ROYAL CIVIL SERVICE COMMISSION
BHUTAN CIVIL SERVICE EXAMINATION (BCSE) 2019
EXAMINATION CATEGORY: TECHNICAL

PAPER III: SUBJECT SPECIALISATION PAPER FOR STATISTICS

Date : October 13, 2019
Total Marks : 100
Writing Time : 150 minutes (2.5 hours)
Reading Time : 15 minutes (prior to writing time)

GENERAL INSTRUCTIONS:

1. Write your Registration Number clearly and correctly on the Answer Booklet.
2. The first 15 minutes is to check the number of pages of Question Paper, printing errors, clarify doubts and to read the instructions. You are NOT permitted to write during this time.
3. This paper consists of **TWO SECTIONS**, namely SECTION A & SECTION B:
   - **SECTION A** has two parts: Part I - 30 Multiple Choice Questions
     Part II - 4 Short Answer Questions

     All questions under SECTION A are COMPULSORY.
   - **SECTION B** consists of two Case Studies. Choose only **ONE** case study and answer the questions of your choice.
4. All answers should be written on the Answer Booklet provided to you. Candidates are not allowed to write anything on the question paper. If required, ask for additional Answer Booklet.
5. All answers should be written with correct numbering of Section, Part and Question Number in the Answer Booklet provided to you. Note that any answer written without indicating the Section, Part and Question Number will NOT be evaluated and no marks will be awarded.
6. Begin each Section and Part in a fresh page of the Answer Booklet.
7. You are not permitted to tear off any sheet(s) of the Answer Booklet as well as the Question Paper.
8. Use of any other paper including paper for rough work is not permitted.
9. **You are required to hand over the Answer Booklet to the Invigilator before leaving the examination hall.**
10. This paper has **12 printed pages**, including this instruction page.

GOOD LUCK

SECTION A

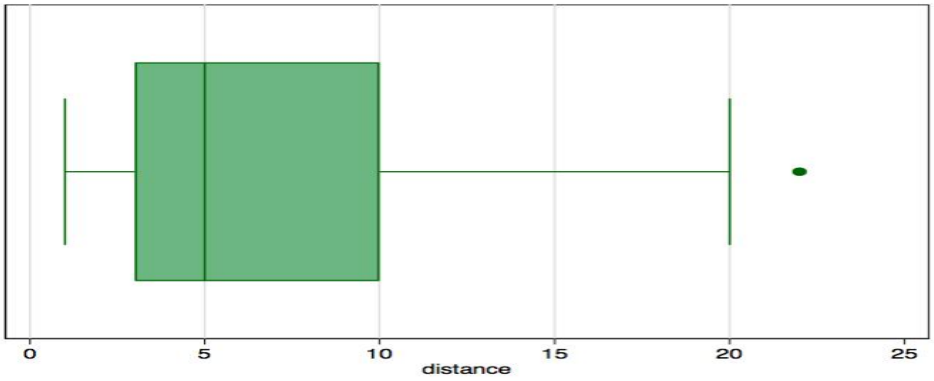
PART I: Multiple Choice Questions (30 marks)

Choose the correct answer and write down the letter of your chosen answer in the Answer Booklet against the question number e.g. 31 (d). Each question carries ONE mark. Any double writing, smudgy answers or writing more than one choice shall not be evaluated.

1. The National Statistics Bureau (NSB) wants to study the use of statistics by civil servants and decides to select 100 civil servants from a list of almost 26,355 regular civil servants as on 31st January, 2019. The 100 civil servants selected in the study is called
   a) Sample
   b) Population
   c) Statistics
   d) Parameter

2. The Director of the Royal Civil Service Commission (RCSC) instructs the statistical officer in the commission to conduct the job satisfaction survey of a total of almost 26,355 regular civil servants in the country. All the civil servants included in the study is called
   a) Sample
   b) Population
   c) Statistics
   d) Parameter

3. The box and whisker plot of distance travelled by students to school is presented below:



   What is the median distance travelled by all students to school?
   a) 5
   b) 10
   c) 15
   d) 20

4. Descriptive statistics consists of procedures used to
   a) summarize the information and describe the characteristics of a set of data.
   b) decide how large a sample should be taken.
   c) make inferences about population characteristics from information contained in a sample.
   d) All of the above.

5. A small corporation employs 100 people. Which descriptive measure would best characterize a typical employee's salary?
   a) Mean
   b) Median
   c) Variance
   d) Mode

6. A disadvantage of the range as a measure of dispersion is
   a) it can be distorted by a large mean.
   b) it is based on only two observations.
   c) it is not in the same units as the original data.
   d) All of the above.

7. The measure of central tendency, which separates the lower 50% of a set of values from the upper 50% of the set is called
   a) Mean
   b) Mode
   c) Median
   d) Inter quartile range

8. Given a frequency distribution that has positive skewness,
   a) the mode will be greater than the median.
   b) the mean will be greater than the median.
   c) the mode will be greater than the mean.
   d) none of the above are necessarily true.

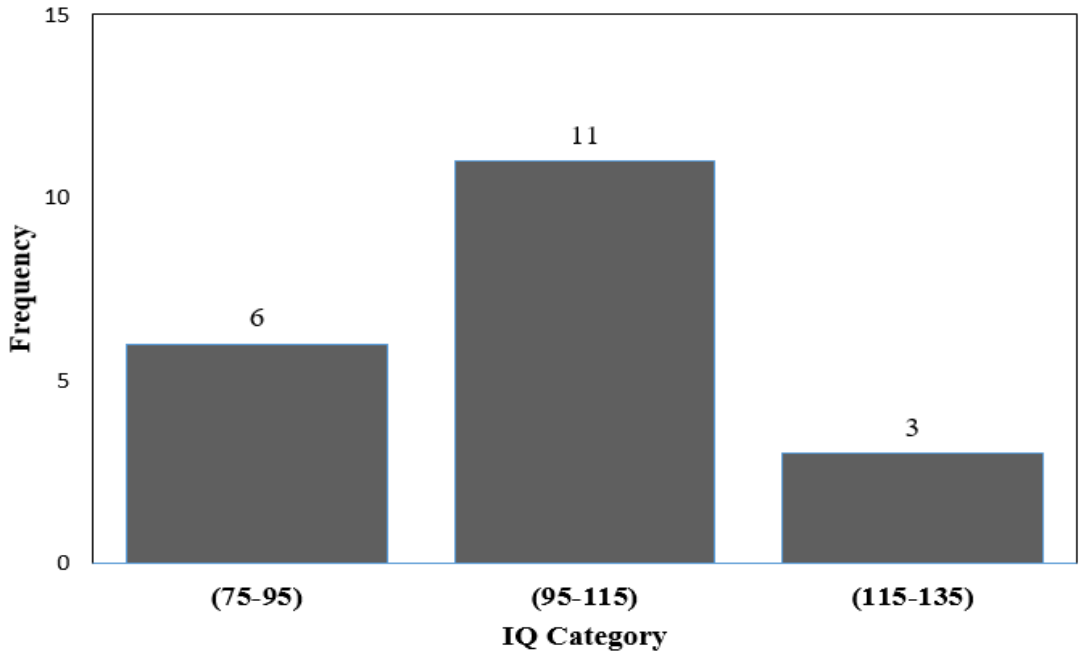
9. If the mean, median and mode of a distribution are 5, 6, 7 respectively, then the distribution is
   a) not skewed.
   b) skewed negatively.
   c) skewed positively.
   d) symmetrical.

10. Which measures of central tendency are least affected when an extremely low or an extremely high value is added to a given set of data?
    a) Mean and median.
    b) Mean and mode.
    c) Mode and median.
    d) None of the above.

11. You are told that a population has a mean of 40 and there is no variance. What can you conclude from this?
    a) All the elements in the population are 40.
    b) There is only one element in the population.
    c) Someone has made a mistake.
    d) None of the above.

Use the following information of the histogram to answer **Questions 12 to 14.**

The IQ of students have been categorized and presented in the histogram below. The IQ of students are categorized into three groups as on x-axis and the number of students in that IQ category are shown on y-axis.



12. What is the total number of students?
    a) 20
    b) 30
    c) 40
    d) 10

13. What is the total number of students with IQ between 75 and 95?
    a) 11
    b) 6
    c) 17
    d) None of the above.

14. What is the total number of students with IQ greater than 95 but less than 135?
    a) 11
    b) 3
    c) 14
    d) None of the above.

Use the following information to answer **Questions 15 and 16.**

The variable X represents the number of times a student visits hospital in a month. The following table provides X and the probabilities of X.

X	0	1	2	3
P(x)	0.1	0.3	0.4	0.2

15. The average or mean number of times a student visits hospital in a month is
    a) 1.5
    b) 0.25
    c) 0.35
    d) 1.70

16. The variance of times a student visits hospital in a month is
    a) 1.67
    b) 0.81
    c) 0.85
    d) 1.25

17. The ages of all boys in the class are 38, 26, 13, 41 and 22. What is the population variance?
    a) 106.8
    b) 91.4
    c) 240.3
    d) 42.4

18. The value of Z_0 such that $P[Z<Z_0] = 0.8531$ is
    a) 0.8023
    b) 0.8051
    c) 1.0500
    d) 0.0500

19. If Z is a standard normal variable, then $P(Z>0)$ is
    a) 0
    b) 0.5
    c) 1.0
    d) None of the above

Use the following information to answer **Questions 20 to 22.**

The scores on a statistics subject are normally distributed with a mean score of 50 and a standard deviation of 15. A sample of 25 students selected from the student population produced a mean score of 55.

20. If a student is selected at random, the probability that the student will obtain a score of 55 or more is
    a) 0.4525
    b) 0.1293
    c) 0.3707
    d) 0.6293

21. From the sample of 25 students, the standard error of the sample mean score is
    a) 0.6
    b) 9.0
    c) 3.0
    d) 15.0

22. From the sample of 25 students, the probability of obtaining a sample mean score of 55 or more is
    a) 0.4525
    b) 0.4515
    c) 0.0475
    d) 0.0485

23. The quality control manager at a light bulb factory needs to estimate that mean life of a large shipment of light bulbs. The population standard deviation is 100 hours. A random sample of 64 light bulbs indicates a sample mean life of 350 hours. What is the 95% confidence interval estimate of the population mean?
    a) (325.5, 374.5)
    b) (329.4, 370.6)
    c) (330.5, 375.5)
    d) (330.0, 375.0)

24. What does the normal distribution look like?
    a) A bell.
    b) A circle.
    c) A square.
    d) A straight line.

25. The total area under the normal distribution curve is equal to
    a) 100
    b) 1
    c) 0
    d) None of the above.

26. Under the 68-95-99.7 empirical rule in a unimodal, symmetric distribution, about 68% of the values fall within how many standard deviations?
    a) 0 standard deviation of the mean.
    b) 1 standard deviation of the mean.
    c) 2 standard deviation of the mean.
    d) 3 standard deviation of the mean.

27. A normal distribution has a mean of 36, and 19% of the values are above 50. What is the standard deviation?
    a) 15.0
    b) 15.91
    c) 16.0
    d) 16.91

Use the following information to answer **Questions 28 to 30.**

The price of a litre of petrol in Thimphu is normally distributed with a mean of Nu. 60 and a standard deviation of Nu. 5. [**hint:** table on z-scores are provided.]

28. What is the probability that at a randomly chosen petrol station in Thimphu, the price of petrol will be between Nu. 60 to Nu. 65?
    a) 0.3159
    b) 0.3643
    c) 0.3413
    d) 0.3643

29. What is the probability that at a randomly chosen petrol station in Thimphu, the price of petrol will be between Nu. 55 and Nu. 60?
    a) 0.3159
    b) 0.3643
    c) 0.3643
    d) 0.3413

30. What percent of the petrol stations have prices between Nu. 55 to Nu. 65?
    a) 0.6000~60.00%
    b) 0.6500~65.00%
    c) 0.6826~68.26%
    d) None of the above.

PART II – Short Answer Questions (20 marks)

This part has 4 Short Answer Questions. Answer ALL the questions. Each question carries 5 marks.

1. Descriptive and inferential statistics; population and sample; parameters and statistics are basic concepts of statistics. How do you differentiate between the following:

    a) Descriptive and inferential statistics

    b) Population and sample

    c) Parameters and statistics

2. The relationship of 200 students between their amount of time spent to study mathematics in minutes a day and the scores on the test are presented in the 2-way table below. Based on the information provided, answer the following questions:

		Study time (in minutes)				
		0-20	**21-40**	**41-60**	**>60**	**Total**
Scores on test (in %)	**80-100**	0	4	16	20	**40**
	60-79	0	20	30	10	**60**
	40-59	2	4	32	32	**70**
	20-39	10	2	8	0	**20**
	0-19	2	0	0	8	**10**
	Total	**14**	**30**	**86**	**70**	**200**

a) What is the number of students who studied more than 60 minutes a day and scored 80-100 percent in the mathematics?

b) What is the percentage of students who scored more than 60% in mathematics?

c) What would be the overall pass percentage in mathematics if the management of school set the pass percentage at 40%?

d) What is the time spent by the majority of students who scored above 80% and by what percentage?

3. Investing in Sonam's Furniture House has 60% chance in gaining Nu 10,000 and almost 40% chance in loosing Nu 3,000. What is the expected value of investing in Sonam's Furniture House?

4. Assume that the mean weight of 1-year-old baby in Bhutan is normally distributed with a mean of 9.5kg and a standard deviation of approximately 1.1 kg. Using the standard empirical rule of normal distribution (68-95-99.7), estimate the percentage of 1-year-old baby that meet the following conditions. Draw a sketch and shade the proper region for each problem.

a) Less than 8.4 kg

b) Between 7.3 kg and 11.7 kg

SECTION B: Case Study (50 marks)

Choose either CASE I or CASE II from this section. Each case study carries 50 marks. Mark for each sub-question is indicated in the brackets.

CASE I

A statistics teacher investigates the relationship between the height and weight of 10 students in a class. The height and weight of 10 students are recorded as follows:

height (cm)	158	162	163	170	154	167	177	170	179	179
weight (kg)	48	57	57	60	45	55	62	65	70	68

The most effective way to display such information is a scatterplot diagram as it provides a visual representation of the relation between the two variables. The visual representation can be even quantified by calculating the sample correlation coefficient (r). In the example above for height and weight of 10 students, calculate the following:

1. Draw a scatterplot showing the relationship between these two variables. (10 marks)
2. Calculate the sample correlation between these two variables. [hint: The statistical formula is provided] (20 marks)
3. Interpret your results. (10 marks)
4. The linear relationship between the height and weight was derived as $ht = 108.74 + 1.01 * wt.$ What would be the estimated height of student if his weight is approximately 80 kg? (10 marks)

Case II

Sonam grows apples in two different fields. When the apples are ready to be picked, he is curious as to whether the sizes of apple plants differ between the two fields. He takes a random sample of plants from each field and measures the heights of the plants. The following is a summary of the results:

Statistics	Field-A	Field-B
Mean	1.3 m	1.6 m
Standard deviation	0.5 m	0.3 m
Number of plants	22	24

Using the above information, you will conduct the two-sample t-test at $\alpha = 0.05$ level of significnace. Answer the following questions:

1. What would be your null and alternative hypothesis? (10 marks)
2. What is the value of t-statistics? (10 marks)
3. What is the approximate P-value? [hint: refer t-distribution table] (20 marks)
4. At $\alpha = 0.05$ level of significance, is there sufficient evidence to conclude that the mean size of apple plants differs between the two fields? (10 marks)

LIST OF STATISTICAL FORMULA

Measures of Central Location and Dispersion:

1. $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
2. $s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n-1)}$
3. $\mu = \frac{\sum_{i=1}^{n} x_i}{N}$
4. $\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{N}$
5. $\mu = E(x) = \sum_{i=1}^{N} x_i P(x_i)$
6. $\sigma^2 = V(x) = \sum_{i=1}^{N} (x_i - \mu)^2 P(x_i) = \sum x_i^2 P(x_i) - \mu^2$
7. $\text{Coefficient of variation (CV)} = \frac{SD}{\bar{x}} * 100$

Normal Distribution:

8. $x \sim \text{Normally}(\mu, \sigma^2)$
9. $z = \frac{x - \mu}{\sigma}$

Confidence Intervals:

10. $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
11. $\bar{x} \pm t_{\frac{\alpha}{2},(n-1)} \frac{s}{\sqrt{n}}$
12. $n = \frac{z^2_{\frac{\alpha}{2}} \sigma^2}{B^2}$

Correlation Co-efficient:

13. $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{(x - \bar{x})^2 (y - \bar{y})^2}}$

t-test:

14. $t - stat = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}$

Table of Normal Curve Areas

The area from the mean to z-score z is given in the table below:

z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4958	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Table of t-distribution critical values

TABLE D

t distribution critical values

	Upper-tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

TASHI DELEK