ROYAL CIVIL SERVICE COMMISSION
BHUTAN CIVIL SERVICE EXAMINATION (BCSE) 2021
EXAMINATION CATEGORY: TECHNICAL

PAPER III: SUBJECT SPECIALISATION PAPER FOR STATISTICS

Date : October 31, 2021
Total Marks : 100
Writing Time : 150 minutes (2.5 hours)
Reading Time : 15 minutes (prior to writing time)

GENERAL INSTRUCTIONS:

1. Write your Registration Number clearly and correctly on the Answer Booklet.
2. The first 15 minutes is to check the number of pages of Question Paper, printing errors, clarify doubts and to read the instructions. You are NOT permitted to write during this time.
3. This paper consists of **TWO SECTIONS**, namely SECTION A & SECTION B:
   - **SECTION A** has two parts: Part I - 30 Multiple Choice Questions
     Part II - 4 Short Answer Questions

     All questions under SECTION A are COMPULSORY.
   - **SECTION B** consists of two Case Studies. Choose only **ONE** case study and answer the questions of your choice.
4. All answers should be written on the Answer Booklet provided to you. Candidates are not allowed to write anything on the question paper. If required, ask for additional Answer Booklet.
5. All answers should be written with correct numbering of Section, Part and Question Number in the Answer Booklet provided to you. Note that any answer written without indicating the Section, Part and Question Number will NOT be evaluated and no marks will be awarded.
6. Begin each Section and Part on a fresh page of the Answer Booklet.
7. You are not permitted to tear off any sheet(s) of the Answer Booklet as well as the Question Paper.
8. Use of any other paper including paper for rough work is not permitted.
9. **You must hand over the Answer Booklet to the Invigilator before leaving the examination hall.**
10. This paper has **13 printed pages**, including this instruction page.

GOOD LUCK

SECTION A

PART I: Multiple Choice Questions (30 marks)

Choose the correct answer and write down the letter of your chosen answer in the Answer Booklet against the question number e.g. 31 (d). Each question carries ONE mark. Any double writing, smudgy answers or writing more than one choice shall not be evaluated.

1. All of the following can bring bias to the statistics, EXCEPT
   a) Selecting the wrong sample from the population.
   b) Designing improper data collection instrument.
   c) Collecting data at odd time.
   d) Uses the non-random process to select the sample from the population.

2. What measures the strength and direction of the linear relationship between two quantitative variables?
   a) Mean
   b) Median
   c) Mode
   d) Correlation

3. Big data concept has recently evolved in Statistical data science. Which of the following statements is NOT TRUE about the Big data?
   a) Big data refers to sets of data that are far too massive to handle with traditional hardware.
   b) Big data is so big in size that statisticians or analysts can't even process, analyse and cannot apply any forecasting techniques.
   c) Big data is gathered on a massive scale by search engines such as Google and social media sites like Facebook and Twitter.
   d) The three characteristics of big data that distinguish from other types of data re volume, velocity and variety.

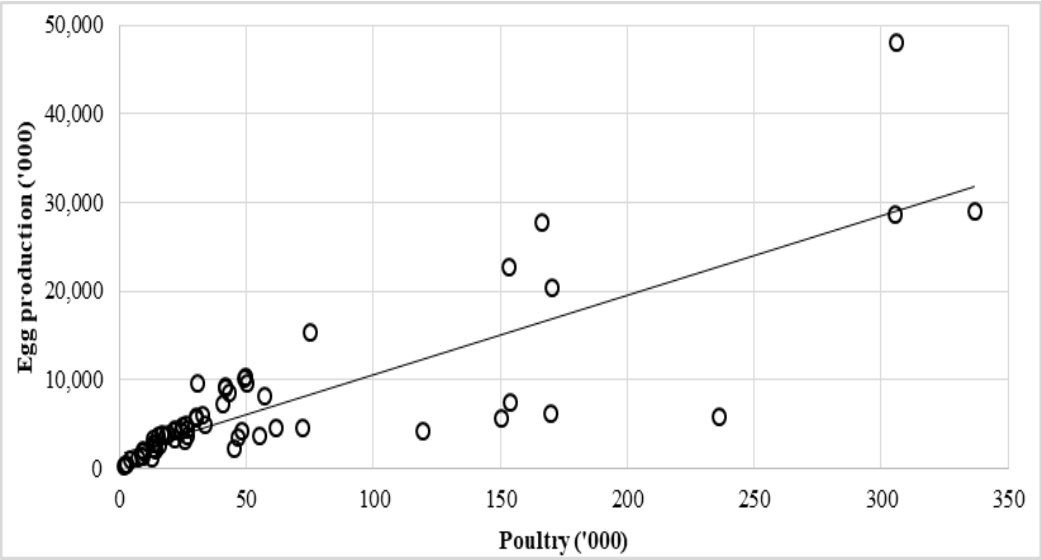
4. All of the following are factors associated with the margin of error in statistics, EXCEPT
   a) The standard deviation of the population
   b) The population size, N
   c) The sample size, n
   d) The level of confidence

5. Before we apply statistical techniques to a dataset, it's important to examine the data to understand its basic properties. We use a series of techniques that are collectively called as *Exploratory Data Analysis* (EDA). Which of the following is NOT the graphical EDA techniques?
   a) Box plots
   b) Histograms
   c) Scatter plots
   d) Hypothesis testing

6. Which of the following is NOT the summary of statistical measures?
   a) Measures of central tendency
   b) Measures of dispersion
   c) Measures of association
   d) Measures of hypothesis testing

7. A set of observations of a single variable collected over time and its analysis is called
   a) Time series analysis
   b) Forecasting techniques
   c) Regression analysis
   d) Data analysis

Use the following scatter plot to answer Question 8. It shows the number of poultry (‘000) and egg production (‘000) by 20 dzongkhags in Bhutan from 2018 to 2020.



8. How do you interpret the relationship between the number of poultry (‘000) and egg production (‘000) in the country?
   a) There is a positive and strong relationship between the number of poultry and egg production.
   b) There is a negative and weak relationship between the number of poultry and egg production.
   c) There is a positive but not so strong relationship between the number of poultry and egg production.
   d) There is no relationship between the number of poultry and egg production.

9. Which of the following is a negatively skewed distribution?
   a) Mean=5, median=6, mode=7
   b) Mean=6, median=5, mode=7
   c) Mean=5, median=5, mode=7
   d) Mean=5, median=5, mode=5

10. Which of the following statistical measures is the robust statistics?
    a) Mean
    b) Median
    c) Mode
    d) Standard deviation

11. Which of the following is NOT TRUE about the basic properties of all probability measures?
    a) All probabilities are not between 0 and 1
    b) The probability of the entire event space is 1
    c) The probability of nothing happening is 0
    d) If two events do not overlap, then the probability of the one or the other happening is the sum of their individual probabilities.

Use the following table to answer **Questions 12 and 13.** It shows the observed and expected returns of a company.

Returns	-15% to -5%	-5% to +5%	+5% to +15%	+15% to +25%
Observed frequency	22	29	37	12
Expected frequency	13.59	34.13	34.13	13.59

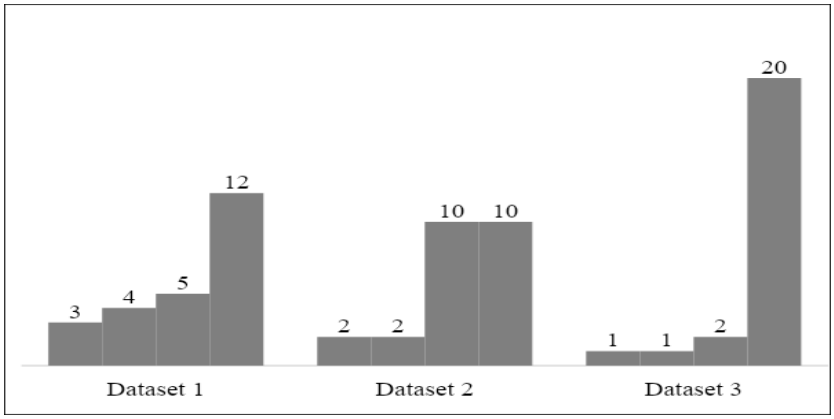
12. What is the test statistics if the returns of the company above follows $\chi 2\ (chi\ square)$ distribution?
    a) 6.10
    b) 6.20
    c) 6.40
    d) 6.60

13. What is the degree of freedom $(k-1-m)$ for the goodness of fit test taken from the $\chi 2\ (chi\ square)$ distribution above?
    a) 2
    b) 3
    c) 4
    d) 5

14. Which of the following show how the geometric mean, harmonic mean and arithmetic mean are related?
    a) $AM > GM > HM$
    b) $AM > GM < HM$
    c) $AM < GM > HM$
    d) $AM = GM = HM$

15. What is the order of mean, median and mode in a negatively skewed distribution?
    a) $Mean < Median > Mode$
    b) $Mean > Median > Mode$
    c) $Mean < Median < Mode$
    d) $Mean = Median = Mode$

Use the bar graph of three datasets given below to answer **Question 16**.



16. Based on the information provided in the bar graph, which of the following is TRUE?
    a) Dataset 3 is more variable compared to Dataset 1 and Dataset 2.
    b) Dataset 1 has more variation compared to Dataset 2 and Dataset 3.
    c) Dataset 2 has more variation compared to Dataset 1 and Dataset 3.
    d) Dataset 1, Dataset 2 and Dataset 3 have equal variation.

17. The Co-efficient of Variation (CV) for product x is 9.3% while for the product y is 8.9%. What can you say about the sale of product x and product y?
    a) Product x is higher
    b) Product y is higher
    c) Product x is lower
    d) Both the product x and y are lower

18. Regardless of difference in the distribution of sample and population, the mean of sampling distribution must be equal to
    a) Degrees of freedom
    b) Population mean
    c) Standard error
    d) Statistics error

19. All of the following are non-random sampling, EXCEPT
    a) Quota sampling
    b) Judgement sampling
    c) Convenience sampling
    d) Stratified sampling

20. Population A has a standard deviation of 3 and a sample size of 8 while population B has a standard deviation of 5 and a sample size of 7. What is the standard deviation of sampling distribution?
    a) 4.044
    b) 3.044
    c) 2.044
    d) 1.044

21. What is the confidence interval if the point estimate is 8 and the margin of error is 5?
    a) 3 to 13
    b) 4 to 14
    c) 5 to 15
    d) 6 to 16

22. What is the formula for calculating confidence interval in confidence interval estimation?
    a) $point\ estimate * margin\ of\ error$
    b) $point\ estimate \pm margin\ of\ error$
    c) $point\ estimate - margin\ of\ error$
    d) $point\ estimate + margin\ of\ error$

23. In stem and leaf display diagram used in exploratory analysis, what is the stem considered as?
    a) Central digits
    b) Trailing digits
    c) Leading digits
    d) Dispersed digits

24. What is the name for cumulative frequency curve?
    a) Ogive
    b) A-give
    c) B-give
    d) C-give

25. In probability theories, the events which can never occur together is classified as
    a) Collectively exclusive event
    b) Collectively exhaustive event
    c) Mutually exhaustive event
    d) Mutually exclusive event

26. A person buys a lottery. His chance of winning a Toyota car is 60% while his chances of winning Hyundai car is 70% and the chance of winning both is 40%. What is his chances of winning Toyota or Hyundai?
    a) 0.50
    b) 0.60
    c) 0.80
    d) 0.90

27. What is the branch of statistics called in which the data is collected according to ordinal scale or nominal scale?
    a) Distribution statistics
    b) Sampling statistics
    c) Parametric statistics
    d) Non-parametric statistics

28. What is the sum of all values if the number of observations is 30 and the value of arithmetic mean is 15?
    a) 15
    b) 45
    c) 200
    d) 450

29. What is the value of geometric mean if the arithmetic mean is 20 and the harmonic mean is 30?
    a) 14.94
    b) 24.94
    c) 34.94
    d) 44.94

30. For Binomial distribution, what is the formula for calculating mean?
    a) $\mu = p + q$
    b) $\mu = np$
    c) $\mu = pq$
    d) $\mu = nq$

PART II – Short Answer Questions [20 marks]

This part has 4 Short Answer Questions. Answer ALL the questions. Each question carries 5 marks.

The following data are the semester tuition charges (‘000 Nu) for a sample of private colleges in the country. At the 0.05 significance level, we can conclude that there is a difference in the mean tuition charges by region.

West (‘000 Nu.)	Central (‘000 Nu.)	East (‘000 Nu.)
10	8	7
11	9	8
12	10	6
10	8	7
12		6

Using the information provided above, answer the following:

1. State the null and alternative hypothesis. (3 marks)
2. What is the decision rule? (2 marks)
3. Develop an ANOVA table. What is the value of test statistics? (10+3 marks)
4. What is the decision regarding the null hypothesis? (2 marks)

SECTION B: Case Study [50 marks]

Choose either CASE I OR CASE II from this section. Each case study carries 50 marks. Mark for each sub-question is indicated in the brackets.

CASE I

A business owner wants to review the relationship between the advertising expenses and sales revenue. The table below shows the information on sales and advertising expense for the last four months.

Month	Advertising Expenses (Million Nu.)	Sales Revenue (Million Nu.)
July	2	7
August	1	3
September	3	8
October	4	10

Using the information provided in the above table, answer the following questions:

1. Which variable is dependent and independent variables? (5 marks)
2. Determine the coefficient of correlation. (20 marks
3. Interpret the strength of the correlation coefficient. (10 marks)
4. Determine the coefficient of determination and interpret. (15 marks)

CASE II

Using the Bhutan Living Standard Survey 2017 data, a researcher wants to study household expenses for energy using the factors like mean household size (hsize) and per capita expense on rent (rent_p). The output of the analysis is as follows:

Source	SS	df	MS		
				Number of obs	= 11,660
				F(2, 11657)	= 606.48
Model	1.0291e+09	2	514531116	Prob > F	= 0.0000
Residual	9.8897e+09	11,657	848387.41	R-squared	= 0.0942
				Adj R-squared	= 0.0941
Total	1.0919e+10	11,659	936505.213	Root MSE	= 921.08

energy_exp	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
hsize	82.53735	4.636933	17.80	0.000	73.44819	91.62652
rent_p	.1873001	.0055635	33.67	0.000	.1763948	.1982054
_cons	177.2497	23.28338	7.61	0.000	131.6104	222.889

Using the information provided in the table above, answer all the questions:

1. Which variable is dependent and independent variables? Determine the regression equation and discuss the regression coefficients. What does it indicate if some coefficients are positive and some coefficients are negative? (20 marks)

2. What is the intercept value and what does it say in the model above? (10 marks)

3. What is the estimated expense for energy at home if the mean household size is four persons and the per capita expense on rent is Nu 10,000 per month? (10 marks)

4. Do you think that the model is rightly specified? What could be some other potential factors for explaining household energy consumption? Explain about three other factors in real life that determine the household energy expenses. (10 marks)

LIST OF STATISTICAL FORMULA

1. $\bar{x} = \frac{\sum_{x=1}^{n} x_i}{n}$

2. $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{(n-1)}$

3. $\mu = \frac{\sum_{i=1}^{n} x_i}{N}$

4. $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{N}$

5. $\mu = E(x) = \sum_{i=1}^{N} x_i P(x_i)$

6. $\sigma^2 = V(x) = \sum_{i=1}^{N}(x_i - \mu)^2 P(x_i) = \sum_{i=1}^{N} x_i{}^2 P(x_i) - \mu^2$

7. $Coefficient\ of\ variation\ (CV) = \frac{SD}{\bar{x}} * 100$

8. $x \sim Normally\ (\mu, \sigma^2)$

9. $z = \frac{x-\mu}{\sigma}$

10. $\underline{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

11. $\underline{x} \pm t_{\frac{\alpha}{2},(n-1)} \frac{s}{\sqrt{n}}$

12. $n = \frac{z^2 \frac{\alpha}{2} \sigma^2}{B^2}$

13. $r = \frac{\sum_{i=1}^{n}(x-\bar{x})(y-\underline{y})}{\sqrt{(x-\bar{x})^2 (y-\bar{y})^2}}$

14. $t - stat = \frac{x_A - x_B}{\sqrt{\frac{s_A{}^2}{n_A} + \frac{s_B{}^2}{n_B}}}$

Table of Normal Curve Areas

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

Table of t-distribution critical values

Upper tail probability values												
df	**0.25**	**0.2**	**0.15**	**0.1**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.0025**	**0.001**	**0.0005**
1	1.000	1.376	1.963	3.078	6.310	12.700	15.900	31.820	63.650	127.300	318.300	636.619
2	0.817	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.080	22.330	31.599
3	0.765	0.979	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.220	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.696	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291
	50%	**60%**	**70%**	**80%**	**90%**	**95%**	**96%**	**98%**	**99%**	**99.50%**	**99.80%**	**99.90%**
Confidence Level												

Table of f-distribution critical values for $\alpha = 0.05$

d2	d1																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9	243.9	245.9	248	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19	19.16	19.25	19.3	19.33	19.35	19.37	19.38	19.4	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.7	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6	5.96	5.91	5.86	5.8	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.5	4.46	4.43	4.4	4.36
6	6.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.1	4.06	4	3.94	3.87	3.84	3.81	3.77	3.74	3.7	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.3	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.5	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.9	2.86	2.83	2.79	2.75	2.71
10	4.96	4.1	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.7	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.2	3.09	3.01	2.95	2.9	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.4
12	4.75	3.89	3.49	3.26	3.11	3	2.91	2.85	2.8	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.3
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.6	2.53	2.46	2.42	2.38	2.34	2.3	2.25	2.21
14	4.6	3.74	3.34	3.11	2.96	2.85	2.76	2.7	2.65	2.6	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.9	2.79	2.71	2.64	2.59	2.54	2.48	2.4	2.33	2.29	2.25	2.2	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.2	2.96	2.81	2.7	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.1	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.9	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.1	2.87	2.71	2.6	2.51	2.45	2.39	2.35	2.28	2.2	2.12	2.08	2.04	1.99	1.95	1.9	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.1	2.05	2.01	1.96	1.92	1.87	1.81
22	4.3	3.44	3.05	2.82	2.66	2.55	2.46	2.4	2.34	2.3	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.8	2.64	2.53	2.44	2.37	2.32	2.27	2.2	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.4	3.01	2.78	2.62	2.51	2.42	2.36	2.3	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.6	2.49	2.4	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.9	1.85	1.8	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.2	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.2	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.7	2.55	2.43	2.35	2.28	2.22	2.18	2.1	2.03	1.94	1.9	1.85	1.81	1.75	1.7	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4	3.15	2.76	2.53	2.37	2.25	2.17	2.1	2.04	1.99	1.92	1.84	1.75	1.7	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.5	1.43	1.35	1.25
Infinity	3.84	3	2.6	2.37	2.21	2.1	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1