ROYAL CIVIL SERVICE COMMISSION BHUTAN CIVIL SERVICE EXAMINATION (BCSE) 2021 EXAMINATION CATEGORY: <u>TECHNICAL</u>

PAPER II: GENERAL SUBJECT KNOWLEDGE PAPER FOR STATISTICS

Date	: October 30, 2021
Total Marks	: 100
Writing Time	: 90 minutes (1.5 hours)
Reading Time	: 15 Minutes (prior to writing time)

GENERAL INSTRUCTIONS:

- 1. Write your Registration Number clearly and correctly on the Answer Booklet.
- 2. The first 15 minutes is to check the number of pages of Question Paper, printing errors, clarify doubts and to read the instructions. You are NOT permitted to write during this time.
- 3. This paper consists of TWO Parts: Part I & Part II

Part I consists of 70 multiple choice questions of 1 (one) mark each, and

Part II consists of short answer questions for 30 marks.

- 4. All questions are COMPULSORY.
- 5. All answers should be written on the Answer Booklet provided to you. Candidates are not allowed to write anything on the question paper. If required, ask for additional Answer Booklet.
- 6. All answers should be written with correct numbering of Part and Question Number in the Answer Booklet provided to you. Note that any answer written without indicating the correct Part and Question Number will NOT be evaluated and no marks will be awarded.
- 7. Begin each Part in a fresh page of the Answer Booklet.
- 8. You are not permitted to tear off any sheet(s) of the Answer Booklet as well as the Question Paper.
- 9. Use of any other paper including paper for rough work is not permitted.
- 10. You must hand over the Answer Booklet to the Invigilator before leaving the examination hall.
- 11. This paper has **19 printed pages**, including this instruction page.

GOOD LUCK!

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Part I

Multiple Choice Questions [70 marks]

Choose the correct answer and write down the letter of your chosen answer in the Answer Booklet against the question number e.g. 71 (a). Each question carries ONE mark. Any double writing, smudgy answers or writing more than one choice shall not be evaluated.

- 1. Which of the following is the science of collecting, organization, presenting, analysing, and interpreting data to assist in making more effective decisions?
 - a) Statistics
 - b) Population
 - c) Sample
 - d) Variable
- 2. The method used to determine something about a population on the basis of a sample is called
 - a) Descriptive statistics
 - b) Inferential statistics
 - c) Population
 - d) Sample
- 3. Which of the following is NOT TRUE about the quantitative variable?
 - a) Quantitative variable is numeric in nature.
 - b) Quantitative variable can either be discrete or continuous variable.
 - c) The information on marital status of an individual is a quantitative variable.
 - d) The amount of air pressure in a vehicle tire is a quantitative variable.
- 4. All of the following properties are about interval-level data, EXCEPT
 - a) Data classifications are mutually exclusive and exhaustive.
 - b) Data classifications are ordered according to the amount of characteristic they possess.
 - c) Data categories have no logical order.
 - d) Equal differences in the characteristic are represented by equal differences in the measurement
- 5. One of the most common ways to portray a frequency distribution is
 - a) Column chart
 - b) Histogram
 - c) Pie chart
 - d) Line chart
- 6. Any measurable characteristic of a population is called
 - a) Variable
 - b) Statistics
 - c) Parameter
 - d) None of the above

- 7. For a set of raw data (ungrouped data), the ______ is the sum of all the sampled values divided by the total number of sampled observations.
 - a) sample mean
 - b) population mean
 - c) sample median
 - d) population median
- 8. Which of the following is NOT TRUE about the characteristics of the arithmetic mean?
 - a) At least the interval scale of measurement is required.
 - b) The sum of deviations from the mean is not equal to 0.
 - c) All the data values are used in the calculation.
 - d) A set of data has only one mean.
- 9. The arithmetic mean of the squared deviations from the mean is called
 - a) Range
 - b) Variance
 - c) Standard Deviation
 - d) Inter-quartile Range
- 10. For any set of observations (sample or population), the proportion of the values that lie within k standard deviations of the mean is at least:
 - a) $1 + \frac{1}{k^2}$
 - b) $1 \frac{1}{k^2}$

c)
$$\frac{1}{k^2}$$

- d) **k**²
- 11. For a symmetrical, belled-shaped frequency distribution, approximately what percentage of the observations lie within plus and minus 1 standard deviation of the mean?
 - a) 68% of the observations
 - b) 95% of the observations
 - c) 99% of the observations
 - d) 99.7% of the observations

Use the following box-and-whisker to answer **Questions 12 to 14.** The box-and-whisker plot shows the consumption of household electricity in 2020 by dzongkhags based on the Bhutan Power Corporation Ltd (BPCL). The consumptions were recorded in Million Ngultrum.



- 12. What was the range of household electricity consumption by dzongkhags?
 - a) About 38 Million Ngultrum
 - b) About 49 Million Ngultrum
 - c) About 80 Million Ngultrum
 - d) About 99 Million Ngultrum
- 13. What was the median household electricity consumption by dzongkhags?
 - a) Approximately 40 MT
 - b) Approximately 780 MT
 - c) Approximately 40 Million Ngultrum
 - d) Approximately 1619MT
- 14. What was the average household electricity consumption by dzongkhags?
 - a) About 12 Million Ngultrum
 - b) About 38 Million Ngultrum
 - c) About 49 Million Ngultrum
 - d) About 80 Million Ngultrum
- 15. Which of the following is the formula for the calculation of geometric mean?

a)
$$GM = \frac{\sum x}{n}$$

b)
$$GM = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n}$$

c)
$$GM = \sqrt[n]{(x_1)(x_2)...(x_n)}$$

d)
$$GM = \frac{\sum |x - \overline{x}|}{n}$$

- 16. Which of the following statement is NOT TRUE about the median?
 - a) At least the ordinal scale of measurement is required.
 - b) It is not influenced by extreme values.
 - c) It is not unique to a set of data.
 - d) Fifty percent of the observations are larger than the median.
- 17. What is the total number of observations if the total sum of square is 20 and the sample variance is 5?
 - a) 4
 - b) 15
 - c) 25
 - d) 35
- 18. According to empirical rule, the standard deviation and mean interval that covers approximately 99.75% of data from a frequency distribution is
 - a) **4μ ± 4σ**
 - b) **3**μ ± 3σ
 - c) $\mu \pm 3\sigma$
 - d) $2\mu \pm 2\sigma$
- 19. Which of the following measure of variation is useful for highly skewed distribution?
 - a) Interquartile deviation
 - b) Quartile deviation
 - c) Interquartile range
 - d) Quartile range
- 20. The standard deviation of data is 12 and the mean is 72. What is the co-efficient of variation?
 - a) 12.67%
 - b) 14.67%
 - c) 16.67%
 - d) 18.67%
- 21. The co-efficient of quartile deviation is 10.34% and the difference between the first and third quartile is 15. What is the sum of third and first quartile?
 - a) 145
 - b) 155
 - c) 165
 - d) 175
- 22. Suppose you received a 5% increase in salary this year and a 15% increase next year. What is the average annual percent increase in salary?
 - a) 9.89
 - b) 10.00
 - c) 1.09
 - d) 1.10

- 23. Kilograms of apple sold by a farmer between 9 am to 3 pm for 5 sample days were: 103, 97, 101, 106 and 103. What is the mean deviation of apple sold by a farmer?
 - a) 102 kg
 - b) 12 kg
 - c) 2.4 kg
 - d) 1.2 kg
- 24. The interquartile range (IQR) is the difference between third (Q3) and first (Q1) quartiles. What is the IQR of marks obtained by students in a class?
 - a) 31
 - b) 50
 - c) 74
 - d) 75

Use the following dot plot to answer **Questions 25 to 27**. It shows the land holding by size in decimals by 21 random households.



- 25. What is the range of land holding in decimals by 21 random individuals?
 - a) 90 decimals
 - b) 50 decimals
 - c) 48 decimals
 - d) 10 decimals

26. How many individuals hold 50 decimals or more?

- a) 4 individuals
- b) 6 individuals
- c) 10 individuals
- d) 15 individuals

- 27. What is the median size of land holding by 21 random individuals?
 - a) 40 decimals
 - b) 48 decimals
 - c) 50 decimals
 - d) 90 decimals
- 28. Which of the following is NOT TRUE about the symmetric set of observations?
 - a) The mean and median are equal.
 - b) The data values are evenly spread around mean and median.
 - c) The data values below the mean and median are the mirror image of those above.
 - d) The data values below the mean and median are not the mirror image of those above.
- 29. What is the range of the coefficient of skewness?
 - a) -3 up to 3
 - b) -1 up to 1
 - c) 0 up to 3
 - d) 0 up to 1

30. A collection of one or more outcomes of an experiment is called

- a) Experiment
- b) Outcome
- c) Event
- d) Probability
- 31. The probability of an event is the number of
 - a) favourable outcomes.
 - b) possible outcomes.
 - c) favourable outcomes divided by the number of possible outcomes.
 - d) favourable outcomes minus the number of possible outcomes.
- 32. Which of the following is TRUE about the small standard deviation for a set of values?
 - a) A small standard deviation for a set of values indicates that the values are close to mean.
 - b) A small standard deviation for a set of values indicates that the values are close to median.
 - c) A small standard deviation for a set of values indicates that the values are close to mode.
 - d) A small standard deviation for a set of values indicates that the values are close to standard deviation itself.
- 33. A mean income for a sample of 50 day-workers in Bhutan is Nu 500 and the standard deviation is Nu 20. According to empirical rule, at least what percent of income of lies between Nu 460 and Nu 540 per day?
 - a) About 68 percent are between Nu 460 and Nu 540
 - b) About 95 percent are between Nu 460 and Nu 540
 - c) About 99.7 percent are between Nu 460 and Nu 540
 - d) About all of the percent (100 percent) are between Nu. 460 and Nu. 540

Use the following diagram to answer **Questions 34 to 36.** It shows the relationship between the number of bearing tress of apples and production in Bhutan according to the Ministry of Agriculture & Forests.



- 34. What is the diagram called
 - a) Scatter plot
 - b) Line chart
 - c) Column chart
 - d) Dot plot
- 35. How many observations were studied?
 - a) 10 observations
 - b) 9 observations
 - c) 8 observations
 - d) 7 observations
- 36. How would you characterize the relationship between the bearing number of apple trees and production?
 - a) There is no relationship between the bearing number of apple trees and production.
 - b) There is a positive relationship between the bearing number of apple trees and production.
 - c) There is a negative relationship between the bearing number of apple trees and production.
 - d) There is very weak relationship between the bearing number of apple trees and production.
- 37. A 95% confidence interval for the unknown population mean (μ) with known sample mean $(\bar{\mathbf{x}})$ of 122 and the population standard deviation (σ) of 20 for a sample size (n) of 25 is
 - a) (114.16,122.00)
 - b) (114.16,120.89)
 - c) (114.16,129.89)
 - d) (122.00,129.89)

Use the following table to answer **Questions 38 to 40**. A survey of graduates in one of the universities in Bhutan revealed the following majors of the students by gender:

Condon				
Gender	Accounting	Management	Finance	Total
Male	100	150	50	300
Female	100	50	50	200
Total	200	200	100	500

38. What is the probability of selecting a female student?

- a) 0.20
- b) 0.30
- c) 0.40
- d) 0.50

39. What is the probability of selecting a Finance or Accounting major?

- a) **0.2**0
- b) **0.4**0
- c) **0.5**0
- d) **0.6**0

40. What is the probability of selecting a female student with Accounting major?

- a) 1.00
- b) 0.50
- c) 0.40
- d) 0.60
- 41. Consider rolling an experiment of six-sided die. What is the probability of the event "an even number of spots appear face up"?
 - a) ¹/₆
 - b) ²/₆
 - c) ³/₆
 - d) $\frac{4}{6}$

Use the following information to answer Questions 42 to 44.

Board of Directors of one of the companies in Bhutan consists of 8 men and 4 women. Consider 4person search committee is to be chosen at random to conduct a nationwide search for a new company President.

42. What is the probability that all four members of the search committee will be women?

- a) 0.5
- b) 0.005
- c) 0.002
- d) 0.001

43. What is the probability that all four members of the search committee will be men?

- a) 0.1414
- b) 0.1515
- c) 0.5050
- d) 0.5000

44. Does the sum of the probabilities for the events described in (42) and (43) equal 1 and why?

- a) Yes, because the sum of the probabilities for events equal 1.
- b) No, there are other possibilities such as three women and one man.
- c) May be, there are possibilities but I'm not sure about it.
- d) I don't know exactly what the question is talking about.

Use the following table to answer **Questions 45 to 47**. Bhutan Hyundai Motors sells new cars and the largest number of cars sold are on Saturday. The following is the probability distribution for the number of cars Bhutan Hyundai Motors expects to sell on a particular Saturday.

Number of Cars sold (x)	Probability (P(x))
0	0.1
1	0.2
2	0.3
3	0.3
4	0.1
Total	1.0

45. What type of distribution is this?

- a) Probability distribution
- b) Discrete probability distribution
- c) Continuous probability distribution
- d) None of the above
- 46. On a typical Saturday, how many new cars does Bhutan Hyundai Motors sell?
 - a) Approximately 4 cars a day
 - b) Approximately 3 cars a day
 - c) Approximately 2 cars a day
 - d) Approximately 1 car a day

47. What is the variance of the distribution?

- a) 1.210
- b) 1.250
- c) 1.290
- d) 1.295

Use the following to answer Questions 48 to 50. For a case where n = 4 and $\pi = 0.60$,

- 48. What is the probability that, x = 2?
 - a) P(x=2) = 0.246
 - b) P(x=2) = 0.346
 - c) P(x = 2) = 0.446
 - d) P(x = 2) = 0.546
- 49. What is the probability that, $x \leq 2$?
 - a) $P(x \le 2) = 0.326$
 - b) $P(x \le 2) = 0.426$
 - c) $P(x \le 2) = 0.526$
 - d) $P(x \le 2) = 0.626$
- 50. What is the probability that, x > 2?
 - a) P(x > 2) = 0.174
 - b) P(x > 2) = 0.274
 - c) P(x > 2) = 0.374
 - d) P(x > 2) = 0.474

Use the following table to answer **Questions 51 to 54**. Bhutan Jazaer Tours and Travel has 7 employees (consider as the population) and the hourly earnings as converted to US\$ from BTN are in the following table:

Employee	Hourly Earnings (US\$)
Sonam	7
Yeshey	7
Sangay	8
Karma	8
Dorji	7
Namgay	8
Kinlay	9

- 51. What is the population mean?
 - a) 9.71
 - b) 8.71
 - c) 7.00
 - d) 7.71

- 52. What is the sampling distribution of the sample mean for a sample of size 2?
 - a) 11 possible samples
 - b) 21 possible samples
 - c) 31 possible samples
 - d) 41 possible samples

53. What is the mean of the sampling distribution?

- a) 4.71
- b) 5.71
- c) 6.71
- d) 7.71
- 54. Which of the following is NOT TRUE about the population mean and the sampling distribution for the information provided above?
 - a) The mean of the sampling distribution is equal to the population mean.
 - b) The dispersion of the sampling distribution of the sample means is narrower than the population distribution.
 - c) The sampling distribution of the sample means tends to become bell-shaped and approximate to the normal probability distribution.
 - d) The mean of the sampling distribution is greater than the population mean.
- 55. A survey is conducted instead of census for all of the following reasons, EXCEPT
 - a) Contacting the entire population is too expensive
 - b) The cost of studying all items in the population is often too expensive
 - c) The sample results are usually inadequate
 - d) It is also impossible physically to check all items in the population

Use the following information to answer Questions 56 to 58.

The Prime Minister wants to know the mean income of a Bhutanese population. A random sample of 256 people reveals a sample mean of Nu 10,000 per month. The sample standard deviation was recorded at Nu 2,050 per month.

56. What is the mean income of the population per month?

- a) The sample mean of Nu 10,000 is the point estimate of the population mean income per month.
- b) In this case, we don't know the population mean income per month. A census is required to conduct.
- c) It is difficult to precisely tell the mean income of the population per month.
- d) None of the above.
- 57. What is the reasonable range of values for the population mean income per month?
 - a) (6,950 to 10,000)
 - b) (7,950 to 12,050)
 - c) (10,000 to 12,050)
 - d) (10,950 to 12,050)

- 58. What does the range of values calculated above for the population mean income per month say?
 - a) The range of values calculated above says the mean income per month for Bhutanese population is too low.
 - b) The population mean income per month calculated above says that the government should raise the minimum pay.
 - c) No matter how many times we select samples of 256 people, perhaps several hundred, we could expect about 95 percent of these confidence intervals to contain the population mean.
 - d) None of the above.
- 59. Which of the following is NOT TRUE about the F-distribution?
 - a) The F-distribution is continuous and it can assume an infinite number of values between zero and positive infinity.
 - b) It is positively skewed and the long tail of the distribution is to the right-hand side.
 - c) It is positively skewed and the long tail of the distribution is to the left-hand side.
 - d) It is asymptotic and as the value of x increases, the F curve approaches the X-axis.
- 60. The probability of rejecting the null hypothesis when it is true is called
 - a) Hypothesis testing
 - b) Null hypothesis
 - c) Level of significance
 - d) Alternative hypothesis
- 61. What is the Type-I Error in hypothesis testing?
 - a) Type I Error is the probability of rejection whether or not the null hypothesis, H_o , is true.
 - b) Type I Error is the probability of acceptance whether or not the null hypothesis, H_o , is true.
 - c) Type I Error is rejecting the null hypothesis, H_o , when it is true.
 - d) Type I Error is rejecting the null hypothesis, H_o , when it is false.
- 62. All of the following are true about the p-value, EXCEPT
 - a) p-value is the probability of observing a sample value as extreme as, or more extreme than, the value observed, given that the null hypothesis is true.
 - b) A p-value is the way to express the likelihood that H_o is false.
 - c) A p-value is the way to express the likelihood that H_o is true.
 - d) If the p-value is small, then it is likely that H_o is false.
- 63. Which of the following is NOT the assumption of ANOVA?
 - a) The population follow the normal distribution.
 - b) The populations are dependent.
 - c) The populations have equal standard deviation.
 - d) The populations are independent.

- 64. What is the total variation in ANOVA test?
 - a) The sum of the squared differences between each observation and the overall (grand) mean.
 - b) The sum of the squared differences between each observation and the treatment mean.
 - c) The sum of the squared differences between each treatment means.
 - d) The sum of the squared differences between each observation .
- 65. When studying characteristics of a population, there are many practical reasons why we prefer to select portions or samples of a population to study. All of the following are reasons for sampling, EXCEPT
 - a) The time to conduct study for the whole population may be prohibitive.
 - b) The cost to conduct study for the whole population may be prohibitive.
 - c) It may be difficult or almost impossible to study the whole population. For example, for the case of population study of fish varieties in Bhutan.
 - d) The sample results are not adequate and we always ignore sample survey.
- 66. In sampling, a sample to be selected from the population has the same chance of being selected. Which of the following is the considered the basis for any sampling method?
 - a) Simple Random Sampling
 - b) Systematic Random Sampling
 - c) Stratified Random Sampling
 - d) Cluster Sampling
- 67. When a population can be clearly divided into groups based on the certain characteristics, the stratified random sampling method is used to ensure that each group is represented in the sample. The selected groups are called:
 - a) Cluster
 - b) Strata
 - c) Group
 - d) All of the above
- 68. If the mean of sampling distribution is equal to population mean, then the sample statistics is often considered:
 - a) Unbiased estimator
 - b) Biased estimator
 - c) Interval estimator
 - d) Hypothesis estimator
- 69. If the standard deviation of the population is 35 and the sample size is 9, which of the following is the standard deviation of sampling distribution?
 - a) 11.67
 - b) 12.67
 - c) 13.67
 - d) 14.67

- 70. In statistical analysis, which of the following is true about the sample size?
 - a) The sample size is considered large if $n \ge 30$
 - b) The sample size is considered large if $n \leq 30$
 - c) The sample size is considered large if $n \ge 50$
 - d) The sample size is considered large if $n \leq 50$

PART II – Short Answer Questions [30 marks]

Answer ALL short answer questions. Marks for each question are indicated in the brackets.

The mean height of a plant is 43 millimetres. A researcher is concerned about the growth of the plants in his garden. He selects a random sample of 12 plants and measures each. The results are reported below in millimetres.

	42	39	42	45	43	40	39	41	40	42	43	42
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- 1. State the null hypothesis and the alternative hypothesis? (3 marks)
- 2. How many degrees of freedom are there? (2 marks)
- 3. Calculate the sample standard deviation. (10 marks)
- 4. Compute the value of t-statistics. (10 marks)
- 5. What is the decision regarding the null hypothesis? Is it reasonable to conclude that the mean growth of a plant is as expected at 0.01 level of significance? (5 marks)

LIST OF STATISTICAL FORMULA

1.
$$\overline{\mathbf{x}} = \frac{\sum_{x=1}^{n} \mathbf{x}_i}{n}$$

2.
$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)}$$

3.
$$\mu = \frac{\sum_{i=1}^{n} x_i}{N}$$

4.
$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{N}$$

5.
$$\mu = E(x) = \sum_{i=1}^{N} x_i P(x_i)$$

6.
$$\sigma^2 = V(x) = \sum_{i=1}^N (x_i - \mu)^2 P(x_i) = \sum_{i=1}^N x_i^2 P(x_i) - \mu^2$$

7. Coefficient of variation (CV) =
$$\frac{SD}{\bar{x}}$$
*100

8.
$$x \sim Normally(\mu, \sigma^2)$$

9.
$$Z = \frac{x-\mu}{\sigma}$$

10.
$$\underline{x} \pm \underline{z} \frac{\alpha}{2} \frac{\sigma}{\sqrt{n}}$$

11.
$$\underline{x} \pm t \underline{\alpha}_{2(n-1)} \frac{s}{\sqrt{n}}$$

$$12. \quad n = \frac{z^2 \frac{\alpha}{2} \sigma^2}{B^2}$$

13.
$$r = \frac{\sum_{i=1}^{n} (x - \bar{x})(y - \underline{y})}{\sqrt{(x - \bar{x})^2 (y - \bar{y})^2}}$$

14.
$$t - stat = \frac{x_A - x_B}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

Table of Normal Curve Areas

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

	Upper tail probability values												
df	0.25	0.2	0.15	0.1	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005	
1	1.000	1.376	1.963	3.078	6.310	12.700	15.900	31.820	63.650	127.300	318.300	636.619	
2	0.817	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.080	22.330	31.599	
3	0.765	0.979	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.220	12.924	
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610	
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869	
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959	
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408	
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041	
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781	
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587	
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437	
12	0.696	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318	
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221	
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140	
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073	
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015	
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965	
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922	
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883	
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850	
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819	
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792	
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768	
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745	
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725	
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707	
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690	
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674	
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659	
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646	
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551	
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496	
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460	
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416	
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390	
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300	
z*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291	
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.50%	99.80%	99.90%	
						Confider	nce Level						

Table of t-distribution critical values

10	d1																		
d2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	161. 4	199. 5	215. 7	224. 6	230. 2	234	236. 8	238. 9	240. 5	241. 9	243. 9	245. 9	248	249. 1	250. 1	251.	252. 2	253. 3	254.
2	18.5	19	19.1 6	19.2 5	19.3	19.3 3	19.3 5	19.3 7	19.3 8	19.4	19.4	19.4 3	19.4 5	19.4 5	19.4 6	19.4 7	19.4 8	19.4 9	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.7	8.66	8.64	8.62	, 8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6	5.96	5.91	5.86	5.8	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.5	4.46	4.43	4.4	4.36
6	6.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.1	4.06	4	3.94	3.87	3.84	3.81	3.77	3.74	3.7	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.3	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.5	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.9	2.86	2.83	2.79	2.75	2.71
10	4.96	4.1	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.7	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.2	3.09	3.01	2.95	2.9	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.4
12	4.75	3.89	3.49	3.26	3.11	3	2.91	2.85	2.8	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.3
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.6	2.53	2.46	2.42	2.38	2.34	2.3	2.25	2.21
14	4.6	3.74	3.34	3.11	2.96	2.85	2.76	2.7	2.65	2.6	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.9	2.79	2.71	2.64	2.59	2.54	2.48	2.4	2.33	2.29	2.25	2.2	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.2	2.96	2.81	2.7	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.1	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.9	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.1	2.87	2.71	2.6	2.51	2.45	2.39	2.35	2.28	2.2	2.12	2.08	2.04	1.99	1.95	1.9	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.1	2.05	2.01	1.96	1.92	1.87	1.81
22	4.3	3.44	3.05	2.82	2.66	2.55	2.46	2.4	2.34	2.3	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.8	2.64	2.53	2.44	2.37	2.32	2.27	2.2	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.4	3.01	2.78	2.62	2.51	2.42	2.36	2.3	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.6	2.49	2.4	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.9	1.85	1.8	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.2	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.2	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.7	2.55	2.43	2.35	2.28	2.22	2.18	2.1	2.03	1.94	1.9	1.85	1.81	1.75	1.7	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4	3.15	2.76	2.53	2.37	2.25	2.17	2.1	2.04	1.99	1.92	1.84	1.75	1.7	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.5	1.43	1.35	1.25
Infinit v	3.84	3	2.6	2.37	2.21	2.1	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1

Table of f-distribution critical values for $\alpha = 0.05$