

ROYAL CIVIL SERVICE COMMISSION
BHUTAN CIVIL SERVICE EXAMINATION (BCSE) 2022
EXAMINATION CATEGORY: TECHNICAL

PAPER III: SUBJECT SPECIALISATION PAPER FOR STATISTICS

Date	: October 9, 2022
Total Marks	: 100
Writing Time	: 150 minutes (2.5 hours)
Reading Time	: 15 minutes (prior to writing time)

GENERAL INSTRUCTIONS:

1. Write your Registration Number clearly and correctly on the Answer Booklet.
2. The first 15 minutes is to check the number of pages of Question Paper, printing errors, clarify doubts and to read the instructions. You are NOT permitted to write during this time.
3. This paper consists of **TWO SECTIONS**, namely SECTION A & SECTION B:
 - **SECTION A** has two parts: Part I - 30 Multiple Choice Questions
Part II - 4 Short Answer Questions
All questions under SECTION A are **COMPULSORY**.
 - **SECTION B** consists of two Case Studies. Choose only **ONE** case study and answer the questions of your choice.
4. All answers should be written on the Answer Booklet provided to you. Candidates are not allowed to write anything on the question paper. If required, ask for additional Answer Booklet.
5. All answers should be written with correct numbering of Section, Part and Question Number in the Answer Booklet provided to you. Note that any answer written without indicating the Section, Part and Question Number will NOT be evaluated and no marks will be awarded.
6. Begin each Section and Part on a fresh page of the Answer Booklet.
7. You are not permitted to tear off any sheet(s) of the Answer Booklet as well as the Question Paper.
8. Use of any other paper including paper for rough work is not permitted.
9. **You must hand over the Answer Booklet to the Invigilator before leaving the examination hall.**
10. This paper has **16 printed pages**, including this instruction page.

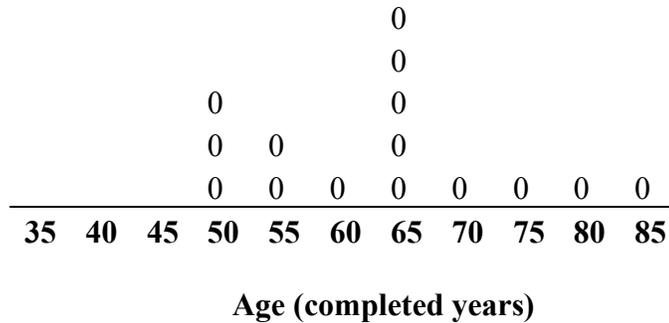
GOOD LUCK

SECTION A

PART I: Multiple Choice Questions (30 marks)

Choose the correct answer and write down the letter of your chosen answer in the Answer Booklet against the question number e.g. 31 (d). Each question carries ONE mark. Any double writing, smudgy answers or writing more than one choice shall not be evaluated.

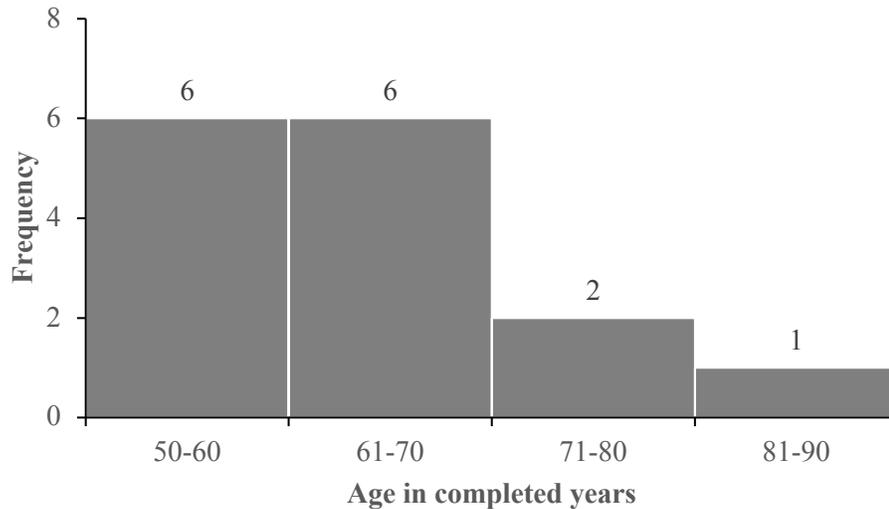
1. The dotplot below displays the total number of students with different completed ages in years.



Which of the following is 50th percentile?

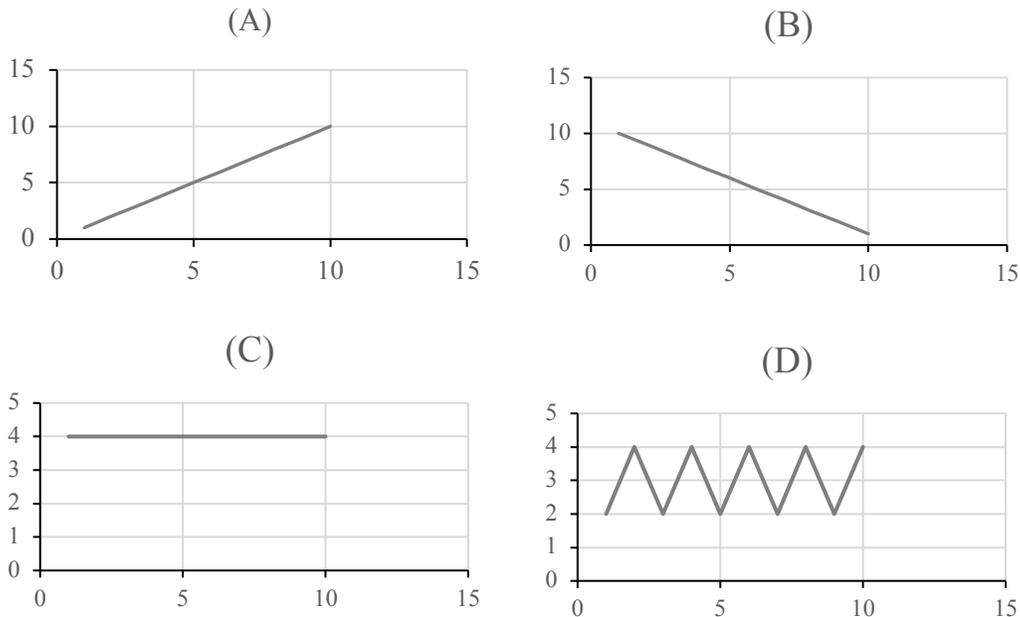
- a) 50 years
 - b) 55 years
 - c) 65 years
 - d) 75 years
2. NSB surveyed a large random sample of people aged 19 to 30 years living in the Thimphu to determine the cost of living expenses and to derive some associated socio-economic indicators. To which of the following populations can the results of this survey be safely generalized?
- a) Only people aged 19 to 30 years living in Thimphu who were involved in the survey
 - b) Only people aged 19 to 30 years living in Thimphu
 - c) All people living in Thimphu
 - d) Only people aged 19 to 30 years living in Bhutan
3. NSB wants to estimate the proportion of defective umbrellas supplied in boxes by one of the suppliers in Thimphu for distribution to census enumerators for data collection. A total of 5,000 umbrellas were supplied in 200 boxes. The store receiver numbers the boxes from 1 to 200 and then randomly selects 6 boxes. She then opens the 6 boxes with the corresponding numbers and examines all 25 umbrellas in each of these boxes, and determines the proportion of the 150 umbrellas that are defectives. What type of sampling approach is used?
- a) Non-random sampling
 - b) Simple random sampling
 - c) Stratified random sampling
 - d) Cluster random sampling

4. Random samples of 15 students were surveyed. In this survey, data on weight were collected on the completed age of the students. The histogram below displays the data collected in the survey.



- In which of the following intervals is the median of these data located?
- 50-60 years' old
 - 61-70 years' old
 - 71-80 years' old
 - 81-90 years' old
5. Test scores of students are being transformed by applying the following rule to each of the raw scores.
 Transformed score = $3.5(\text{raw score}) + 6.2$
 Which of the following statements is **NOT TRUE**?
- The mean transformed score equals $3.5(\text{the mean raw score}) + 6.2$
 - The median transformed score equals $3.5(\text{the median raw score}) + 6.2$
 - The range of the transformed score equals $3.5(\text{the median raw score}) + 6.2$
 - The standard deviation of the transformed score equals $3.5(\text{the standard deviation of the raw scores})$
6. Which one of the following is **NOT** the summary of statistical measures?
- Measures of central tendency
 - Measures of dispersion
 - Measures of association
 - Measures of hypothesis testing
7. A researcher wants to test the null hypothesis that a car is not defective. Under which of the following conditions would a type-1 error would be committed?
- The researcher concludes that the car is not defective when it actually is not.
 - The researcher concludes that the car is not defective when it actually is.
 - The researcher concludes that the car is defective when it actually is.
 - A type-1 error cannot be committed in this situation.

8. The following scatterplots are the association between the number of students and the time spent for preparing maths exam.



Using the information provided above, which one of the following scatterplots represent a data set with a correlation coefficient of $r = -1$?

- Scatterplot A
 - Scatterplot B
 - Scatterplot C
 - Scatterplot D
9. Based on a random sample of 50 Royal Thimphu College (RTC) students, the 90 percent confidence interval for the mean amount of money students spent per meal on lunch in the canteen is found to be (100, 150). Which of the following statements is **TRUE**?
- 90% of the time, the mean amount of money that all students spend on lunch at the canteen will be between Nu 100 and Nu 150.
 - 90% of all students spend between Nu 100 and Nu 150 on lunch at this high school.
 - 90% of the random sample of 50 students surveyed at RTC would result in a sample mean amount of money students spend on lunch between Nu 100 and Nu 150.
 - 90% of the random sample of 50 students surveyed at RTC would result in a 90% confidence interval that contains the true mean amount of money students spend on lunch.

10. An outlier may be defined as a data point that is more than 1.5 times the interquartile range below the lower quartile or is more than 1.5 times the interquartile range above the upper quartile. The following are the data on the weights of 15 students in a school in kilogram.

50	60	65	55	70	75	80	85	50	65	65	65	50	55	65
----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

- According to the definition of the outlier, how many students can we treat as outlier with respect to the completed age?
- a) There are 0 student that we can treat as outlier
b) There are 5 students that we can treat as outlier
c) There are 6 students that we can treat as outlier
d) There are 10 students that we can treat as outlier
11. Which of the following is the best way to decrease the margin of error?
- a) Increase the confidence level to 99%
b) Increase the sample size
c) Decrease the sample size
d) Use the population standard deviation

Use the following data pairs (x, y) to answer **Question 12** and **13**.

(1, 1.24), (2, 5.23), (3, 7.24), (4, 7.60), (5, 9.97), (6, 14.31), (7, 13.99), (8, 14.88), (9, 18.04), (10, 20.70)

12. What is the regression equation?
- a) $y=0.490x-0.053$
b) $y=2.04x$
c) $y=1.98x+0.436$
d) $y=0.49x$
13. What is the value of the correlation coefficient?
- a) $r=0.490$
b) $r=0.985$
c) $r=0.971$
d) $r=0.240$
14. For some statistical representation, the median is used instead of the mean. Which of the following statements is **TRUE**?
- a) The mean is affected by the skewness, whereas the median is not.
b) The median is always the preferred statistics.
c) The mean will be less than the median when the data are strongly skewed to the right.
d) The mean should be used only when data are strongly skewed to the left.

15. Researchers want to compare the mean time taken to complete maths test by students. 20 randomly selected students who sat for maths test took a mean of 2.5 minutes and a standard deviation of 0.8 minute. Another 20 different randomly selected students who sat for maths test took a mean of 2.1 minutes and a standard deviation of 1.1 minutes. Assuming that the conditions for inference are met, which of the following statements about the p-value obtained from the data and the conclusion of the significance test is **TRUE**?
- a) The p-value is less than 0.01; therefore, there is a significant difference in mean time taken to complete the maths test.
 - b) The p-value is greater than 0.01 but less than 0.05; therefore, there is a significant difference in mean time taken to complete the maths test.
 - c) The p-value is greater than 0.05 but less than 0.10; therefore, there is a significant difference in mean time taken to complete the maths test.
 - d) The p-value is greater than 0.10; therefore, there is no significant difference in mean time taken to complete the maths test.
16. The Population and Housing Census of Bhutan (PHCB) was last conducted in 2017 to gather key information for the purposes of policy formulation, socio-economic planning, service delivery, and other indicators for measuring progress towards the achievement of key government targets. Which government agency in Bhutan is responsible for conducting nationwide population and housing census?
- a) Department of Civil Registration and Census, Ministry of Home and Cultural Affairs
 - b) National Statistics Bureau
 - c) Ministry of Labour and Human Resources
 - d) HM's Secretariat
17. What is the median if the mean and mode of the data set are respectively 4 and 10?
- a) 1.5
 - b) 5.3
 - c) 6
 - d) 16
18. If the standard deviation of data 0,1,2,3,4,5,6,7,8,9 is 'k', what would be the standard deviation of data 10,11,12,13,14,15,16,17,18,19?
- a) k
 - b) k+1
 - c) k+4
 - d) k+8
19. The mean of four numbers is 37. The mean of the smallest three of them is 34. If the range of the data is 15, what is the mean of the largest three?
- a) 41
 - b) 40
 - c) 39
 - d) 38

20. What is the value of variance given the mean of the data is 100 and the coefficient of variation is 45%?
- 2025
 - 450
 - 45
 - 4.5

21. What is the value of k given the mode of the following data is 7?

3	8	6	7	1	6	10	6	7	$2k+5$	9	7	13
---	---	---	---	---	---	----	---	---	--------	---	---	----

- 1
 - 3
 - 4
 - 7
22. The mean and the variance of 10 observations are 4 and 2 respectively. What will be the new mean and variance, if every observation is multiplied by 2?
- (8, 4)
 - (8, 8)
 - (8, 20)
 - (80, 40)
23. In the regression equation $y = a + bx$, what is the variable y called?
- Independent variable
 - Dependent variable
 - Constant
 - Intercept
24. If the size of the population is large, what is the expected size of the sample?
- Smaller
 - Larger
 - Accurate
 - Fixed
25. A statistician obtained the mean and the standard deviation of 100 observations as 40 and 5.1. Later, it was observed that one observation was wrongly entered as 50 instead of the correct figure 40. What are the corrected mean and the standard deviation?
- (39.9, 6)
 - (39.9, 5)
 - (39.9, 4)
 - (39.9, 3)

26. The table below gives the probability distribution of the random variable x .

x	8	12	16		20	24
$p(x)$	$1/8$	$1/6$	$3/8$	$1/4$		$1/12$

What is the variance of the random variable x ?

- a) 20
 - b) 21
 - c) 22
 - d) 23
27. What is the relationship among the mode, median and mean for a symmetric distribution?
- a) Mode=2 median-3 mean
 - b) Mode=3 median-4 mean
 - c) Mode=3 median-2 mean
 - d) Mode=2 median- 4 mean
28. Based on the best sampling practices, what would be the order of the sampling schemes from best to worst?
- a) Simple random, stratified, convenience
 - b) Simple random, convenience, stratified
 - c) Stratified, simple random, convenience
 - d) Stratified, convenience, simple random
29. The table below provides the number of children and families in a survey conducted to estimate the mean household size of the family.

Number of children	Number of families
0	8
1	16
2	22
3	14
4	6
5	4
6	2

Using the information provided above, what is the mean number of children per family?

- a) 1.91
- b) 2.19
- c) 2.47
- d) 3.14

30. For a right skewed distribution, what is the relationship between the mean and median?
- a) $mean = median$
 - b) $mean \approx median$
 - c) $mean < median$
 - d) $mean > median$

PART II – Short Answer Questions [20 marks]

This part has 4 Short Answer Questions. Answer ALL the questions. Each question carries 5 marks.

The table below shows correlation between the variables read, write, math and science scores of students received on these tests. The variable female is a 0/1 variable coded 1 if the student was female and 0 otherwise. The STATA syntax and output of the correlation between these variables are provided.

corr read write math science female

(obs =200)

	read	write	math	science	female
read	1.0000				
write	0.5968	1.0000			
math	0.6623	0.6174	1.0000		
science	0.6302	0.5704	0.6307	1.0000	
female	-0.0531	0.2565	-0.0293	-0.1277	1.0000

Using the information provided above, answer the following:

1. What is the total number of observations that were used in the correlations? Is there any missing observation? **(5 Marks)**
2. The correlation between read and read is 1.0000. What does that mean? **(5 Marks)**
3. What is the correlation between read and write? What does the value indicate? **(5 Marks)**
4. What is the correlation between read and female? **(5 Marks)**

SECTION B: Case Study [50 marks]

Choose either CASE I OR CASE II from this section. Each case study carries 50 marks. Mark for each sub-question is indicated in the brackets.

CASE I

Below is the descriptive information on the score students received on a writing test including the STATA command and the annotated output.

summarize write, detail

writing score					

Percentiles		Smallest			
1%	31	31			
5%	35.5	31			
10%	39	31	Obs		200
25%	45.5	31	Sum of Wgt.		200
50%	54		Mean		52.775
			Std. Dev.		9.478586
		Largest			
75%	60	67			
90%	65	67	Variance		89.84359
95%	65	67	Skewness		-.4784158
99%	67	67	Kurtosis		2.238527

Using the information provided above, answer all the questions.

1. What is the value of the first quartile and what does that value indicate? (10 marks)
2. What is the value of the median? What does that value indicate? Is median a good measure of central tendency? If yes, when and why? (15 marks)
3. What is the range of the writing test score? What is the mean writing test score? What can you say about the data distribution? (10 marks)
4. What do you understand by skewness and kurtosis? What can you say about the data distribution by referring to the values of skewness and kurtosis? (15 marks)

CASE II

For survey data analysis, most people do not conduct their own surveys. Rather, they use survey data that some agency has already collected and made available to the public. It is very important to know what kind of sampling design was used to collect the data.

Answer all the questions with regard to the sampling designs.

1. What do you understand by the probability weight? Give an example to elaborate on the probability weight. Should the probability weights be equal to the population total? **(15 marks)**
2. What do you understand by primary sampling unit (PSU)? Give an example **(10 marks)**
3. In the sampling design, there is also a concept on stratification. What is stratification in the sampling? Give an example. What is the purpose of stratification? **(10 marks)**
4. What is non-response weight? What can be done to improve non-response in survey? **(10 marks)**
5. Explain what do you know about the sampling with and without replacement? **(5 marks)**

TASHI DELEK

LIST OF STATISTICAL FORMULA

n = sample size

N = population size

f = frequency

Σ = sum

w = weight

Sample mean: $\bar{x} = \frac{\Sigma x}{n}$

Population mean: $\mu = \frac{\Sigma x}{N}$

Weighted mean: $\bar{x} = \frac{\Sigma(w * x)}{\Sigma x}$

Sample standard deviation: $s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$

Population standard deviation: $\sigma = \sqrt{\frac{\Sigma(x - \mu)^2}{N}}$

Sample variance: $s^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1}$

Population variance: $\sigma^2 = \frac{\Sigma(x - \mu)^2}{N}$

Limits for Unusual Data: $\mu \pm 2 \sigma$

Empirical Rule:

About 68%: $\mu \pm 1 \sigma$

About 95%: $\mu \pm 2 \sigma$

About 99.7%: $\mu \pm 3 \sigma$

Sample Coefficient of Variation: $CV = \frac{s}{\bar{x}} * 100\%$

Sample z - score: $z = \frac{x - \bar{x}}{s}$

Interquartile Range: $IQR = Q_3 - Q_1$

Boxplot Outliers = $Q_1 \pm 1.5 * (IQR)$

Mean of a discrete probability distribution: $\mu = \Sigma [x * p(x)]$

Standard deviation of a probability distribution: $\sigma = \sqrt{\Sigma [x^2 * p(x)] - \mu^2}$

Binomial Distributions

$r = \text{number of success (or } x)$

$p = \text{probability of success}$

$q = \text{probability of failure}$

$$q = 1 - p; p + q = 1$$

$$P(r) = nC_r p^r q^{n-r}$$

$$\text{Mean: } \mu = np$$

$$\text{Standard deviation: } \sigma = \sqrt{npq}$$

Sample size for estimating:

$$\text{Means: } n = \left(\frac{Z_{\alpha/2} * \sigma}{E} \right)^2$$

$$\text{Proportions: } n = \hat{p}\hat{q} \left(\frac{Z_{\alpha/2} * \sigma}{E} \right)^2$$

Regression and Correlation:

$$\text{Linear correlation coefficient (r): } r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n(\sum x^2) - (\sum x)^2} \sqrt{n(\sum y^2) - (\sum y)^2}}$$

Table of Normal Curve Areas

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

Table of t-distribution critical values

df	Upper tail probability values											
	0.25	0.2	0.15	0.1	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.310	12.700	15.900	31.820	63.650	127.300	318.300	636.619
2	0.817	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.080	22.330	31.599
3	0.765	0.979	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.220	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.696	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.50%	99.80%	99.90%
Confidence Level												

Table of f-distribution critical values for $\alpha = 0.05$

d2	d1																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9	243.9	245.9	248	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19	19.16	19.25	19.3	19.33	19.35	19.37	19.38	19.4	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.7	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6	5.96	5.91	5.86	5.8	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.5	4.46	4.43	4.4	4.36
6	6.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.1	4.06	4	3.94	3.87	3.84	3.81	3.77	3.74	3.7	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.3	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.5	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.9	2.86	2.83	2.79	2.75	2.71
10	4.96	4.1	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.7	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.2	3.09	3.01	2.95	2.9	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.4
12	4.75	3.89	3.49	3.26	3.11	3	2.91	2.85	2.8	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.3
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.6	2.53	2.46	2.42	2.38	2.34	2.3	2.25	2.21
14	4.6	3.74	3.34	3.11	2.96	2.85	2.76	2.7	2.65	2.6	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.9	2.79	2.71	2.64	2.59	2.54	2.48	2.4	2.33	2.29	2.25	2.2	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.2	2.96	2.81	2.7	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.1	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.9	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.1	2.87	2.71	2.6	2.51	2.45	2.39	2.35	2.28	2.2	2.12	2.08	2.04	1.99	1.95	1.9	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.1	2.05	2.01	1.96	1.92	1.87	1.81
22	4.3	3.44	3.05	2.82	2.66	2.55	2.46	2.4	2.34	2.3	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.8	2.64	2.53	2.44	2.37	2.32	2.27	2.2	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.4	3.01	2.78	2.62	2.51	2.42	2.36	2.3	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.6	2.49	2.4	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.9	1.85	1.8	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.2	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.2	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.7	2.55	2.43	2.35	2.28	2.22	2.18	2.1	2.03	1.94	1.9	1.85	1.81	1.75	1.7	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4	3.15	2.76	2.53	2.37	2.25	2.17	2.1	2.04	1.99	1.92	1.84	1.75	1.7	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.5	1.43	1.35	1.25
Infinitely	3.84	3	2.6	2.37	2.21	2.1	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1