

**ROYAL CIVIL SERVICE COMMISSION
BHUTAN CIVIL SERVICE EXAMINATION (BCSE) 2022
EXAMINATION CATEGORY: TECHNICAL**

PAPER II: GENERAL SUBJECT KNOWLEDGE PAPER FOR STATISTICS

Date	: October 8, 2022
Total Marks	: 100
Writing Time	: 90 minutes (1.5 hours)
Reading Time	: 15 Minutes (prior to writing time)

GENERAL INSTRUCTIONS:

1. Write your Registration Number clearly and correctly on the Answer Booklet.
2. The first 15 minutes is to check the number of pages of Question Paper, printing errors, clarify doubts and to read the instructions. You are NOT permitted to write during this time.
3. This paper consists of **TWO Parts: Part I & Part II**
Part I consists of 70 multiple choice questions of 1 (one) mark each, and
Part II consists of short answer questions for 30 marks.
4. All questions are COMPULSORY.
5. All answers should be written on the Answer Booklet provided to you. Candidates are not allowed to write anything on the question paper. If required, ask for additional Answer Booklet.
6. All answers should be written with correct numbering of Part and Question Number in the Answer Booklet provided to you. Note that any answer written without indicating the correct Part and Question Number will NOT be evaluated and no marks will be awarded.
7. Begin each Part in a fresh page of the Answer Booklet.
8. You are not permitted to tear off any sheet(s) of the Answer Booklet as well as the Question Paper.
9. Use of any other paper including paper for rough work is not permitted.
- 10. You must hand over the Answer Booklet to the Invigilator before leaving the examination hall.**
11. This paper has **21 printed pages**, including this instruction page.

GOOD LUCK!

Part I

Multiple Choice Questions [70 marks]

Choose the correct answer and write down the letter of your chosen answer in the Answer Booklet against the question number e.g. 71 (a). Each question carries ONE mark. Any double writing, smudgy answers or writing more than one choice shall not be evaluated.

- Which one of the following is **TRUE** about the Statistics?
 - It is the science of collecting data to assist in making more effective decisions.
 - It is the science of collecting and organizing data to assist in making more effective decisions.
 - It is the science of collecting, organizing and presenting data to assist in making more effective decisions.
 - It is the science of collecting, organizing, presenting, analysing and interpreting data to assist in making more effective decisions.
- 240 adults and children attended a show organized by Film Association of Bhutan in Thimphu. The ratio of adults to children was 5 to 1. How many children attended the show?
 - 40
 - 48
 - 192
 - 200
- Every month Sonam reads two new books and add them to her library. Which of the following types of functions best models the number of books in Sonam's library as a function of time?
 - Increasing exponential
 - Decreasing exponential
 - Increasing linear
 - Decreasing linear
- In Maths test, the mean score is between 3 and 4. The large majority of scores are 3 and 4 with only a few scores of 0, 1, 2 and 5. In English test, the mean score is 2.5, and scores are evenly distributed across all possible scores, with many scores not close to the mean score. Which of the following is **TRUE** about the Maths and English test?
 - The standard deviation of test scores in Maths is less than that of test scores in English.
 - The standard deviation of test scores in Maths is greater than that of test scores in English.
 - The standard deviation of test scores in Maths is equal to that of test scores in English.
 - There is no enough information to compare the standard deviations.
- A researcher selects 100 students at random from a school and finds that the mean weight of 65 kg with an associated margin of error of 10 kg. Which of the following is the most appropriate conclusion based on the data?
 - All the students randomly chosen that day have a weight between 35 kg and 165 kg.
 - All the students have a weight between 35 kg and 165 kg.
 - It is plausible that the mean weight of students randomly chosen that day is between 35 kg and 165 kg.
 - It is plausible that the mean weight of students in schools is not between 35 kg and 165 kg.

6. A school offers a Dzongkhag course. This year, all students in the course were offered additional grammar lessons. The students who took additional grammar lessons did better in the course than students who didn't take the additional grammar lessons. Based on these results, which of the following is the most appropriate conclusion?
- a) Taking additional grammar lessons will cause an improvement for any student who takes any foreign language course.
 - b) Taking additional grammar lessons will cause an improvement for any student who takes dzongkhag course.
 - c) Taking additional grammar lessons was the cause of the improvement for the students who took Dzongkhag course.
 - d) No conclusion about cause and effect can be made regarding students who took the additional grammar lessons and their performance in the Dzongkhag course.
7. What is the process called if you are using information from a sample to make generalization about the larger population?
- a) Statistical inference
 - b) Statistical information
 - c) Statistical data
 - d) None of the above
8. The average age of the students in the class is 22 years. Which of the following is **TRUE** about the statement?
- a) The statement describes inferential statistics.
 - b) The statement describes descriptive statistics.
 - c) The statement does not describe inferential nor descriptive statistics.
 - d) The statement gives no detailed information.
9. What is the mean, median and mode of the observations: 9,6,10,8,10,8,4,8,3,8?
- a) 7.1, 8, 7
 - b) 7.4, 8, 8
 - c) 7.5, 8, 0
 - d) 7, 8, 8
10. Which of the following is **NOT TRUE** about the descriptive statistical problem?
- a) Information revealed in a data set is summarised.
 - b) Predictions are made about a larger set of data.
 - c) Data are displayed visually in graphs.
 - d) Patterns in the data set are identified.
11. Wangmo secured 89 on her history exam and 81 on her maths exam. The mean score for history paper was 82 with a standard deviation of 5 while for maths the mean score was 76 with a standard deviation of 3. On which exam did she do better compare to the rest of the class?
- a) She did better on her maths exam.
 - b) She did better on her history exam.
 - c) Her scores for maths and history were same although the mean scores and standard deviations were different.
 - d) None of the above

Use the following information to **answer Questions 12, 13 and 14.**

Assume that the heights of Statistics Graduates of 2022 are normally distributed, with a mean of 65 inches and standard deviation of 2.5 inches.

12. What percentage of Statistics graduates are taller than 65 inches?
 - a) 50%
 - b) 25%
 - c) 20%
 - d) 10%

13. What percentage of Statistics graduates are between 62.5 inches and 67.5 inches?
 - a) 50%
 - b) 65%
 - c) 68%
 - d) 75%

14. What percentage of Statistics graduates are between 60 inches and 70 inches?
 - a) 50%
 - b) 68%
 - c) 95%
 - d) 100%

Questions 15, 16 and 17. Given the following z-scores on a standard normal distribution, find the percent of the total area under the standard normal curve.

15. What percent is the area to the left of $z=1.45$?
 - a) 50.65%
 - b) 68.65%
 - c) 92.65%
 - d) 99.65%

16. What percent is the area to the right of $z=-0.13$?
 - a) 45.00%
 - b) 50.00%
 - c) 55.17%
 - d) 56.00%

17. What percent is the area between $z=1.37$ and $z=2.98$?
 - a) 5.39%
 - b) 7.39%
 - c) 8.39%
 - d) 8.49%

18. National Statistics Bureau wants to conduct a study to estimate area under paddy cultivation in the country. The sample was randomly chosen from Renewable Natural Resources (RNR) Census data of 2019. What is the variable of interest in this study?
 - a) All paddy growers in the western regions.
 - b) All paddy growers in the central regions.
 - c) All paddy growers in the southern and eastern regions.
 - d) All paddy growers in the country irrespective of their locations.

19. There were 60 Dzongkha graduates, 56 Maths graduates, 40 Bio-science graduates and x statistics graduates sitting for PE coaching class provided for free in Thimphu. The corresponding relative frequency for Dzongkha is 0.3, Maths is 0.3, Bio-science is 0.2 and statistics is 0.2. How many statistics graduates were there in the PE coaching?
- 52
 - 48
 - 44
 - 22

Use the stem-and-leaf plot to answer **Question 20**.

A survey was conducted in the civil service to determine how subordinates feel about the leadership quality of their supervisor. Subordinates were asked to rate the overall quality from 0 (no quality at all) to 100 (extremely good quality).

Stem	Leaf
3	2 9
4	0 3 4 7 8 9 9 9
5	0 1 1 2 3 4 5
6	1 2 5 6 6
7	1 9
8	
9	6

20. What percentage of the subordinates rated their supervisor as very good leadership (ratings 80 and above)?
- 1 %
 - 4%
 - 6%
 - 24%
21. Which of the following is **NOT** a measure of central tendency?
- Mean
 - Median
 - Mode
 - Range

Use the following table to answer **Question 22**. It shows the summary statistics of the high school dropout rates in 1998 and 2002.

	Drop 1998	Drop 2002
N	51	51
Mean	28.15	26.75
Median	27.43	25.65

22. Which of the following statements is **TRUE** about the shape of the distribution of the high school dropout rates in 1998 and 2002?
- Both the 1998 and 2002 high school dropout rates have distributions that are skewed to the right.
 - The 1998 high school dropout rate has distribution skewed to the left and the 2002 has distribution skewed to the right.
 - The 1998 high school dropout rate has distribution skewed to the right and the 2002 has distribution skewed to the left.
 - None of the above
23. Which of the following statements is **TRUE** about the mean and median?
- The mean and median are useful measures of central tendency for both qualitative and quantitative data.
 - The mean and median are useful measures of central tendency for only qualitative data.
 - The mean and median are useful measures of central tendency for only quantitative data.
 - The mean and median are not useful measures of central tendency for both qualitative and quantitative data.
24. The following are the standardized test scores of 5 students in the class with a mean score of 500 points and a standard deviation of 100 points.

Nima: 575	Dawa: 690	Karma: 750	Sonam: 280	Wangmo: 440
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Using the information provided above, which of the students have scores within the 2 standard deviations of the mean?

- Karma, Sonam
- Nima, Dawa, Wangmo
- Nima, Dawa
- Nima, Dawa, Karma, Wangmo

Use the following observation to answer Questions 25, 26 and 27.

21	9	7	6	5	2	2	0
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25. What is the range of the observations?
- 21
 - 9
 - 7
 - 6

26. What is the interquartile range of the observations?
- 21
 - 9
 - 7
 - 6
27. What is the standard deviation of the observations? Consider these observations as population.
- 5.69
 - 6.14
 - 6.50
 - 6.54

28. The summary statistics of 1000 randomly sampled taxis in Thimphu with the total number of passengers carried in a year 2021 are provided below.

Min	4,005	Q1	5,605
Max	10,605	Q3	8,605
Ave	7,005	Std. Dev.	1,400

Using the information provided above, what is the percentage of taxis in Thimphu with the total number of passengers between 5,605 and 8,605?

- 100%
 - 75%
 - 50%
 - 25%
29. The weights of pregnant women were measured in kg. The mean weight was obtained with 83 kg and the standard deviation was 5 kg. Which of the following weights would be classified as an outlier?
- 73 kg
 - 78 kg
 - 91 kg
 - 99 kg
30. Which of the following statements is **TRUE** about the presence of outlier in the data?
- The measurement may be correct and from the same population as the rest but represent a rare event.
 - The measurement belongs to a population different from that from which the rest of the same was drawn.
 - The measurement is incorrect. It may have been observed, recorded, or entered incorrectly.
 - All of the above are explanations for outliers
31. The lower quartile of the weights of the pregnant women was recorded at 88 kg. Which of the following statements is **TRUE**?
- 25% of the pregnant women's weights were 88 kg.
 - 75% of the pregnant women's weights were less than 88 kg.
 - 75% of the pregnant women's weights were greater than 88 kg.
 - 88% of the pregnant women's mean weight was 88 kg.

32. A, B, C and D are sample points and the only possible outcomes of an experiment. The probabilities of the sample points are provided in the table below.

Sample point	A	B	C	D
Probability	1/8	1/8	1/8	

Using the information provided above, what is the probability of the sample point D?

- a) $\frac{1}{8}$
 - b) $\frac{3}{8}$
 - c) $\frac{5}{8}$
 - d) $\frac{1}{4}$
33. An experiment was conducted. Two dices were rolled and the resulting values were summed. Which of the following is NOT a sample point for this experiment?
- a) 1
 - b) 2
 - c) 6
 - d) 7

Use the following table to answer **Questions 34, 35 and 36**. It shows the types of medals offered to winning participants of the national open championship by Bhutan Olympic Committee (BoC).

Gold	Gold	Silver	Gold	Bronze	Silver	Silver
Bronze	Gold	Silver	Silver	Bronze	Silver	Gold
Gold	Silver	Silver	Bronze	Bronze	Gold	Silver
Gold	Gold	Bronze	Bronze			

34. What is the sample points for the above championship?
- a) {Gold, Silver, Bronze}
 - b) {Gold, Gold, Silver, Bronze}
 - c) {Gold, Silver, Silver, Bronze}
 - d) {Gold, Silver, Bronze, Bronze}
35. What is the probability of offering the gold medal to the winning participants of the national open championship by BoC?
- a) 0.25
 - b) 0.33
 - c) 0.36
 - d) 0.50

36. What is the probability of offering the gold and silver medals to the winning participants of the national open championship by BoC?
- a) 0.50
 - b) 0.66
 - c) 0.72
 - d) 0.75
37. There are two bags containing sweets. Bag A contains 72 sweets with an estimated total mass of 852.4g while bag B contains 24 sweets with an estimated total mass of 282.8g. Using the information provided, what would be the mean mass of the sweets altogether?
- a) 15.767 g
 - b) 11.783 g
 - c) 11.839 g
 - d) 11.825 g

Use the following information to answer **Questions 38, 39 and 40.**

A survey of graduates in one of the universities in Bhutan revealed that the sum of the ages of 10 boys was 127 years in 5 years ago while the sum of 15 girls in the next 5 years' time will be 351 years.

38. What is the mean age of the 10 boys today?
- a) 7.7 years
 - b) 10.7 years
 - c) 17.7 years
 - d) 27.7 years
39. What is the mean age of the 15 girls today?
- a) 28.4 years
 - b) 18.4 years
 - c) 11.4 years
 - d) 10.4 years
40. What is the mean age of the 10 boys and 15 girls combine together?
- a) 17.70 years
 - b) 18.12 years
 - c) 18.40 years
 - d) 28.40 years

Use the table below to answer **Question 41**. The marks obtained for monthly Maths test by two students are given.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Total Marks
Sonam	55	75	80	55	75	80	65	70	555
Dorji	90	60	80	50	85	60	55	75	555

41. Which of the following statements is **NOT TRUE** about these students' performance in the monthly Maths test?
- The mean score in the monthly Maths test by Sonam and Dorji is the same
 - Sonam is a consistent performer compared to Dorji in the monthly Maths test
 - The patterns of the marks obtained by Dorji do not vary much compared to Sonam
 - Nothing can be said about Sonam and Dorjis' performance in the monthly Math test

Use the following information to answer **Questions 42, 43 and 44**.

Every Saturday, Dorji invites his sister Pema to the cinema or to the lunch. 70% of his invitations are to the cinema and 90% of the these are accepted. Pema rejects 40% of her brothers' invitation to the lunch.

42. What is the probability that Dorji invites his sister Pema to the lunch?
- 0.70
 - 0.50
 - 0.30
 - 0.20
43. What is the probability that Pema rejects her brother Dorjis' invitation to the lunch?
- 0.12
 - 0.13
 - 0.14
 - 0.15
44. What is the probability that Pema rejects her brother Dorjis' invitation to the cinema?
- 0.02
 - 0.04
 - 0.05
 - 0.07

Use the following table to answer **Questions 45, 46 and 47**. It shows the probability distribution for X.

x	0	5	15	20
P(X=x)	1/12	3/12	5/12	3/12

45. What is the expectation of the distribution?
- 11.5
 - 12.5
 - 13.5
 - 14.5

46. What is the variance of the distribution?
 a) 41.75
 b) 42.75
 c) 43.75
 d) 44.75
47. What is the standard deviation of the distribution?
 a) 6.46
 b) 6.54
 c) 6.61
 d) 6.69
48. The weight of a newborn baby recorded by the JDWNRH, Thimphu was found to be normally distributed with mean 3.35 kg and variance 0.085 kg². How many of the 1,356 babies born last year had weight of less than 3.5 kg?
 a) 943 newborn babies
 b) 843 newborn babies
 c) 743 newborn babies
 d) 643 newborn babies
49. Given that $X \sim N(11, 25)$, what is the probability that $P(X < 18)$?
 a) $P(x < 18) = 0.619$
 b) $P(x < 18) = 0.719$
 c) $P(x < 18) = 0.819$
 d) $P(x < 18) = 0.919$
50. Given that $X \sim N(20, 7)$, what is the probability that $P(X \leq 16.6)$?
 a) $P(x \leq 16.6) = 0.0994$
 b) $P(x \leq 16.6) = 0.0884$
 c) $P(x \leq 16.6) = 0.0774$
 d) $P(x \leq 16.6) = 0.0664$

Use the following to answer **Questions 51, 52 and 53**. The stem plot of the recorded ages of 20 students by gender in a school is provided below.

Male	Female
	1 5 5
5 0	2 0 5 5
5 0	3 0 0 5
5 0	4 0
5 0	5
0	6

51. Which of the following statements is **TRUE**?
 a) The median age of male is less than that of female
 b) The median age of male is almost equal to that of female
 c) The median age of male is greater than that of female
 d) The median age of male is 25 years while that of female is 40 years

52. What is the range of ages for male and female?
- Male=40 years; Female=25 years
 - Male=40 years; Female=26 years
 - Male=40 years; Female=27 years
 - Male=40 years; Female=28 years
53. Which of the following statements is **TRUE**?
- The mean age of male is equal to the median age of female.
 - The mean age of male is equal to the median age of male.
 - The mean age of female is equal to the median age of male.
 - The mean age of female is equal to the median age of female.

Use the information provided in the table below to answer **Question 54**.

Parameter	Estimate	t-Statistics	Prob> t	SE
Intercept	3.415	4.932	0.0004	0.613
No. of mulching cows	0.36	0.84	0.041	0.028

54. Which of the following is the best fit regression line?
- $y=0.36+3.415x$
 - $y=3.415+3.6x$
 - $y=3.415+0.36x$
 - $y=4.932+0.84x$
55. Suppose a simple random sample of size 50 is selected from a population. Which of the following is **TRUE** about the sample?
- It is selected so that every set of 50 subjects in the population has an equal chance of being the sample chosen.
 - It is not drawn in such a way that every subject has the same chance of being selected.
 - Some members of the population have no chance of being selected, but those that can be selected have the same change of being selected.
 - None of the members have equal chance of being selected.

Use the following table to answer **Questions 56, 57 and 58**. An Accountant in National Statistics Bureau wants to investigate the association between the Gross Domestic Product (GDP) and the population in the country.

Year	GDP (million Nu.)	Population (Million)
2009	61,220.56	0.68
2010	72,496.64	0.70
2011	84,950.01	0.71
2012	97,452.96	0.72
2013	105,378.35	0.73
2014	119,545.75	0.75
2015	132,140.72	0.76
2016	149,151.78	0.73
2017	164,627.92	0.74

56. What is the value of the linear correlation coefficient?

- a) 0.50
- b) 0.65
- c) 0.75
- d) 0.78

57. The output of the regression analysis is provided below based on the study conducted by the national accountant.

Source	SS	df	MS	Number of obs=9	
Model	6.19E+09	1	6.19E+09	F(1, 7)	= 12.37
Residual	3.50E+09	7	500110159	Prob > F	= 0.0098
Total	9.69E+09	8	1.21E+09	R-squared	= 0.6387
				Adj R-squared	= 0.5871
				Root MSE	= 22363

GDP	Coefficient	Std. err.	t	P>t	[95% conf. interval]	
Population	1110086	315562.1	3.52	0.01	363899.8	1856272
_cons	-694532.7	228728.7	-3.04	0.019	-1235390	-153675.2

Using the information provided in the table above, what is the equation of the regression line?

- a) $\hat{y} = -694533 + 228729x$
- b) $\hat{y} = -694533 + 1110086x$
- c) $\hat{y} = 228729 + 315562x$
- d) $\hat{y} = -3.04 + 3.52x$

58. What will be the value of GDP when population is close to 1 million in 2025?

- a) 465,804.00 million
- b) 415,553.30 million
- c) 544,290.80 million
- d) 0.48 million

59. Which of the following is **TRUE** about the Central Limit Theorem for a sample size n ?

- a) A standard deviation of the sample mean random variable is greater than the population standard deviation.
- b) The expectation of the sample mean random variable is equal to the population mean μ when n is large.
- c) The sample distribution of the sample mean is always normal for any sample size n .
- d) The standard deviation of the set of sample mean random variable is equal to the population standard deviation σ .

60. A 95% confidence interval for the difference (treatment – control) in the mean activity levels were 2.5 and 4.7. Which of the following is **TRUE** about this interval?

- a) We are 95% confident that the average decrease in activity level in the sample is 3.6 hours.
- b) We are 95% confident that the increase in the mean decrease lies between 2.5 and 4.7
- c) The activity level of 95% decreased between 2.5 and 4.7
- d) We are 5% confident that the increase in the mean decrease lies between 2.5 and 4.7

61. A 95% confidence interval for the difference (treatment – control) in the mean activity levels were 2.5 and 4.7. What will happen to the confidence interval if a 90% confidence interval is applied instead?
- Confidence interval would be wider and involve a larger risk of being incorrect.
 - Confidence interval would be wider and involve a smaller risk of being incorrect.
 - Confidence interval would be narrower and involve a larger risk of being incorrect.
 - Confidence interval would be narrower and involve a smaller risk of being incorrect.
62. For the case of income distribution study, central tendency measure such as that of median is used instead of others. Which of the following statements is **TRUE**?
- The income distributions generally have unequal class intervals.
 - The income distributions are generally symmetrical.
 - The income distributions are generally bimodal.
 - Income distributions are generally skewed.
63. Which of the following best describes the data?
- For the case of left skewed distribution data, mean is less than median.
 - For the case of right skewed distribution data, mean is less than median.
 - For the case of symmetric distribution data, mean is greater than median.
 - For the case of symmetric distribution data, mean is greater than median.
64. The correlation coefficient between the variables x and y is -0.94 . Which of the following statements is **TRUE**?
- x causes y
 - y causes x
 - low scores on x are associated with high scores on y
 - low scores on x are associated with low scores on y
65. The average height of men is 71 inches with a standard deviation of 4 inches, while the average height of women is 66 inches with a standard deviation of 3 inches. What will be the mean and standard deviation for the difference in heights if we look at pairing up one man with one woman?
- Mean of 5 inches with a standard deviation of 3.5 inch
 - Mean of 5 inches with a standard deviation of 1 inch
 - Mean of 5 inches with a standard deviation of 5 inch
 - Mean of 68.5 inches with a standard deviation of 1 inch
66. Sonam says “There is a strong positive correlation between the number of firefighters at a fire and the amount of damage it does. So sending more firefighters just causes more damage.” Which of the following statements is **TRUE** about the conclusion made by Sonam?
- The conclusion drawn is incorrect which is an example of a confounding variable.
 - The conclusion drawn is correct as it is an example of causation.
 - The conclusion drawn is incorrect as it is an example of common response.
 - The conclusion drawn is correct although it is influenced by a lurking variable.

Use the table below to answer **Questions 67 and 68**. It shows the probability distribution of two characteristics: age and traffic violations in the last 12 months.

Age	None	One	Two+
Under 18	0.23	0.12	0.05
18 or older	0.45	0.14	0.01

67. What is the probability that a randomly selected had no traffic violations in the last 12 months given that he/she is 18 or older?
- a) 0.45
 - b) 0.60
 - c) 0.65
 - d) 0.75
68. What is the probability of being under 18 or having one violation?
- a) 0.12
 - b) 0.26
 - c) 0.54
 - d) 0.66
69. A distribution which is skewed to the right has mean 19 and mode 16. What would be value of median?
- a) 14
 - b) 18
 - c) 20
 - d) 25
70. Which of the following measures will be useful in deciding if a group of individuals varies more with respect to height or varies more with respect to weight?
- a) Inter-quartile range
 - b) Standard deviation
 - c) Coefficient of variation
 - d) Correlation coefficient

PART II – Short Answer Questions [30 marks]

Answer ALL short answer questions. Marks for each question are indicated in the brackets.

The following are the monthly prices of a random sample of 2-Bedded houses in hundreds in Thimphu. Using the information provided below, answer the following questions.

7	9	6	6	8	8	8	8	6	9	7	5
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1. Is this a sample or a population? **(5 marks)**
2. What is the range of the monthly prices for 2-bedded houses in thousands in Thimphu? **(5 marks)**
3. What is the mean monthly price for 2-bedded houses in thousands in Thimphu? **(5 marks)**
4. What can you say about the median monthly price for 2-bedded houses in thousands in Thimphu? **(5 marks)**
5. Calculate the sample standard deviation using the given monthly prices of 2-bedded houses in thousands in Thimphu. **(10 marks)**

TASHI DELEK

LIST OF STATISTICAL FORMULA

n = sample size

N = population size

f = frequency

Σ = sum

w = weight

Sample mean: $\bar{x} = \frac{\Sigma x}{n}$

Population mean: $\mu = \frac{\Sigma x}{N}$

Weighted mean: $\bar{x} = \frac{\Sigma(w * x)}{\Sigma x}$

Sample standard deviation: $s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$

Population standard deviation: $\sigma = \sqrt{\frac{\Sigma(x - \mu)^2}{N}}$

Sample variance: $s^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1}$

Population variance: $\sigma^2 = \frac{\Sigma(x - \mu)^2}{N}$

Limits for Unusual Data: $\mu \pm 2 \sigma$

Empirical Rule:

About 68%: $\mu \pm 1 \sigma$

About 95%: $\mu \pm 2 \sigma$

About 99.7%: $\mu \pm 3 \sigma$

Sample Coefficient of Variation: $CV = \frac{s}{\bar{x}} * 100\%$

Sample z - score: $z = \frac{x - \bar{x}}{s}$

Interquartile Range: $IQR = Q_3 - Q_1$

Boxplot Outliers = $Q_1 \pm 1.5 * (IQR)$

Mean of a discrete probability distribution: $\mu = \sum [x * p(x)]$

Standard deviation of a probability distribution: $\sigma = \sqrt{\sum [x^2 * p(x)] - \mu^2}$

Binomial Distributions

$r =$ number of success (or x)

$p =$ probability of success

$q =$ probability of failure

$q = 1 - p; p + q = 1$

$P(r) = nC_r p^r q^{n-r}$

Mean: $\mu = np$

Standard deviation: $\sigma = \sqrt{npq}$

Sample size for estimating:

Means: $n = \left(\frac{Z_{\alpha/2} * \sigma}{E}\right)^2$

Proportions: $n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2} * \sigma}{E}\right)^2$

Regression and Correlation:

Linear correlation coefficient (r): $r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n(\sum x^2) - (\sum x)^2} \sqrt{n(\sum y^2) - (\sum y)^2}}$

Table of Normal Curve Areas

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

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Table of t-distribution critical values

df	Upper tail probability values											
	0.25	0.2	0.15	0.1	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.310	12.700	15.900	31.820	63.650	127.300	318.300	636.619
2	0.817	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.080	22.330	31.599
3	0.765	0.979	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.220	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.696	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.50%	99.80%	99.90%
Confidence Level												

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Table of f-distribution critical values for $\alpha = 0.05$

d2	d1																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9	243.9	245.9	248	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19	19.16	19.25	19.3	19.33	19.35	19.37	19.38	19.4	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.7	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6	5.96	5.91	5.86	5.8	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.5	4.46	4.43	4.4	4.36
6	6.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.1	4.06	4	3.94	3.87	3.84	3.81	3.77	3.74	3.7	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.3	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.5	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.9	2.86	2.83	2.79	2.75	2.71
10	4.96	4.1	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.7	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.2	3.09	3.01	2.95	2.9	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.4
12	4.75	3.89	3.49	3.26	3.11	3	2.91	2.85	2.8	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.3
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.6	2.53	2.46	2.42	2.38	2.34	2.3	2.25	2.21
14	4.6	3.74	3.34	3.11	2.96	2.85	2.76	2.7	2.65	2.6	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.9	2.79	2.71	2.64	2.59	2.54	2.48	2.4	2.33	2.29	2.25	2.2	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.2	2.96	2.81	2.7	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.1	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.9	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.1	2.87	2.71	2.6	2.51	2.45	2.39	2.35	2.28	2.2	2.12	2.08	2.04	1.99	1.95	1.9	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.1	2.05	2.01	1.96	1.92	1.87	1.81
22	4.3	3.44	3.05	2.82	2.66	2.55	2.46	2.4	2.34	2.3	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.8	2.64	2.53	2.44	2.37	2.32	2.27	2.2	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.4	3.01	2.78	2.62	2.51	2.42	2.36	2.3	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.6	2.49	2.4	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.9	1.85	1.8	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.2	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.2	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.7	2.55	2.43	2.35	2.28	2.22	2.18	2.1	2.03	1.94	1.9	1.85	1.81	1.75	1.7	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4	3.15	2.76	2.53	2.37	2.25	2.17	2.1	2.04	1.99	1.92	1.84	1.75	1.7	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.5	1.43	1.35	1.25
Infinity	3.84	3	2.6	2.37	2.21	2.1	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1